Computing and Measuring Nonlinear and Linear Propagation by Two Independent Methods

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Nonlinear effects, caused by propagation of ultrasonic pulses with finite amplitudes, were computed and measured in water in the case of pulses with pressures up to 1.5 MPappp
used in diagnostic devices. An electronic transmitter generated high (280 Vpp) and low
(47 Vpp) voltages, applied to a plane PZT transducer causing in this way nonlinear and linear propagation effects. The carrier frequency of the pulse was 2 MHz, while its time duration was 2.5 ms. The measurements were carried out by means of a typical calibrated PVDF membrane hydrophone and by an electromagnetic (EM) hydrophone, prepared for this study. The pulse measurements by means of the PVDF hydrophone showed a higher number of spectral components than those by means of the EM hydrophone. This effect was explained by sensitivity characteristics that increased in the PVDF and decreased in the EM hydrophone as a function of frequency. Previously, it was shown that the effective frequency band used in measurements by means of the PVDF hydrophone is situated below the resonance, on the increasing slope of the resonance curve. The properties of the EM hydrophone were analyzed on the basis of the plane wave assumption. A procedure was developed to correct distortions of the pulse spectrum and its pressure measured by PVDF and EM hydrophones. In the first case the maximum peak-to-peak pulse pressure should be decreased by 27%, while in the second case it should be increased by only 0.7%, and by 3% if an additional amplifier was used. The sensitivities of PVDF and EM hydrophones were very different and equal for the frequency of 2 MHz to 28 mV/MPa and 0.10 mV/MPa, respectively. The calibration of the EM hydrophone was carried out by means of only two simple: electrical and magnetic independent measurements, although in the EM hydrophone there occurred external interfering signals. For the theoretic-numerical determination of the acoustic fields and their spectra generated in the case of nonlinear and linear propagation the numerical procedure called the WJ Code was applied. It was developed recently by the last-named author of this paper. In calculations absorption in water was taken into account. The critical distance, where distortions caused by nonlinear propagation in water were maximum, was determined by a number of computations of the ultrasonic field as a function of the distance from the transducer. A good agreement between computed results and those measured by two different methods, showing the pulse pressure distribution along the whole beam axis, was confirmed. In this case it was shown that the λ/4 matching layer covering the transducer surface influenced the edge wave radiated by the transducer.

Keywords: ultrasound, nonlinear propagation, pulses, diagnostics, hydrophone
1. Introduction

When investigating nonlinear effects caused by propagation of finite amplitude disturbances in fluids and in soft tissues, it is sometimes necessary to measure pressure pulses with spectral distributions up to frequencies of 20 MHz or, even, higher. This is the case typical of ultrasonography where the pulses of short time duration, in the MHz range, with amplitudes equal or higher than 1 MPa, are sometimes applied [3]. The problem of exact measurements of nonlinear distortions is crucial for nonlinear acoustics, so we decided to investigate this problem in more detail. The purpose of this paper is to show and to discuss distortions of nonlinear effects caused by measurements with PVDF hydrophones and to present a different measurement method by means of an electromagnetic (EM) hydrophone that seems to be more exact. At the same time, we would like to compare the experimental results with numerical ones obtained by means of the WJ Code developed recently by the last-named author [19].

2. Experimental equipment

The principle of the experimental system used in measurements is shown in Fig. 1. In these experiments we used a 2 MHz PZT plane transducer, 1 cm in radius, coated with a \( \lambda/4 \) matching layer. To obtain nonlinear propagation we applied a pulse transmitter with an output of 280 \( V_{pp} \), while for linear propagation a voltage of 47 \( V_{pp} \) was used. The measurements during nonlinear and linear propagation were carried out by switching the transmitter without changing the probe. In this way it was possible to compare directly the nonlinear and linear effects in the same medium, on the same wave path and almost at the same time. The pulse duration time equaled 2.5 \( \mu s \).

![Diagram](image-url)

Fig. 1. The system used in measurements: T — transmitting PZT transducer, W — water, H — PVDF hydrophone, E — its active electrode, EM — electromagnetic hydrophone, C — its sensitive gold conductor, B — ultrasonic beam axis.
All the measurements were carried out at a temperature of 22°C, in a water container connected to a microscopic positioning table. The electronic equipment consisted of a switchable pulse transmitter and an additional calibrated amplifier. The output signals from hydrophones were recorded by the LeCroy 9450A digital storage oscilloscope. It allowed up to 50000 points with 400 MHz sampling frequency to be recorded. Data was stored in the memory of the RAM card. The single shot data was recorded to a 8 bit accuracy. Averaged data and FFT frequency spectrum of the signal were stored with a 16 bit accuracy. After measurements data stored in the memory card was transferred into the PC computer via the GP-IB interface (General Purpose Interface Bus). Data was converted in the computer from the LeCroy binary format (type × dkb) to the text format and was converted by the software written in the Turbo Pascal language.

For first, approximate pressure measurements a laboratory made PVDF hydrophone was used with the sensitivity of 18 mV/MPa. However, for exact pressure measurements the PVDF bilaminar membrane hydrophone (Model 804-041 without preamplifier) was used. It was produced by Sonic Technology (Hatboro, USA) and calibrated at the National Physical Laboratory (Teddington, England). Its active gold electrode was 0.6 mm in diameter and the sensitivity equaled 28 mV/MPa at a frequency of 2 MHz. The calibrated sensitivity characteristic is shown in Table 1 as a function of frequency.

<table>
<thead>
<tr>
<th>Frequency [MHz]</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>14</th>
<th>16</th>
<th>18</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sensitivity [mV/MPa]</td>
<td>28</td>
<td>33</td>
<td>37</td>
<td>41</td>
<td>44</td>
<td>47</td>
<td>50</td>
<td>53</td>
<td>55</td>
<td>56</td>
</tr>
</tbody>
</table>

**Table 1.** Calibrated sensitivity of the PVDF hydrophone.

![Graph](image_url)

**Fig. 2.** The calculated modulus of the frequency characteristic of the system water — gold layer — perspex for various thickness of gold (Au) and aluminium (Al).
To obtain independent information about pressure pulses measurements were also carried out by means of an EM hydrophone [13]. This type of the hydrophone was used for absolute measurements of ultrasonic pulses [8], then for diagnostic ultrasound [4], and also recently for shock wave measurements [6]. For the present study a special EM hydrophone was prepared with a gold conductor, 0.3 mm in width, 1.3 mm in length, and 0.5 μm thick. For such a case frequency characteristics of the system composed of water-gold conductor-perspex block, used in this hydrophone, were computed in the plane wave approximation [6], giving the result shown in Fig. 2.

3. Pressure at the source

The crucial problem of our investigations was the determination of the pressure amplitude at the surface near the front surface of the radiating transducer. Its value is decisive for the numerical description of nonlinear distortions in the acoustic field. In Fig. 3 are presented distributions of the peak-to-peak pressure measured by the PVDF hydrophone in water at the distance of $z = 8$ mm from the transducer. These distributions show some irregularities that may be partially caused by vibrations of several modes of the piezoelectric ceramic transducer [14]. Knowing the maximum measured pressure near the transducer equal to $p_0 = 0.97$ MPa$_{pp}$ and its distribution across the beam it was possible to determine boundary conditions of the acoustic pressure at the transducer surface (see the next section).

![Diagram](Fig. 3. Radial distributions of the acoustic pressure measured by the PVDF hydrophone in water in front of the radiating transducer at the transmitter voltage of 280 V$_{pp}$ (solid line and points) and of 47 V$_{pp}$ (dashed line and points). Vertical axis shows the measured voltage at the hydrophone output and calculated pressures. The dashed line represents an approximate curve $f(r) = p_0[1 - 0.8(|r/a|)^{2.5}]$ used in calculations.)
Near the transducer surface no nonlinearities caused by propagation effects in water were expected. However, we observed in our case at the transducer surface a fundamental frequency of 2 MHz and additionally the third harmonic which was generated by the transmitter-transducer set (Fig. 4, top). Due to the improper frequency characteristic of the PVDF hydrophone the measured amplitude of the third harmonic was increased by about 30% (see Table 1). It was interesting to notice that the third harmonic decreased with the distance $z$, for example at $z = 50$ cm the measured amplitude was already comparable with other harmonics which were generated due to nonlinear propagation in water (Fig. 4, bottom). Therefore, in the first approximation the third harmonic at source was neglected.

![Graphs showing pressure pulses and their spectra](image)

Fig. 4. Pressure pulses and their spectra measured by the PVDF hydrophone for the transmitter voltage of $280 \text{ V}_{\text{pp}}$ at the distance of $z = 8$ mm (top) and at the distance of $z = 50$ mm (bottom).

4. Numerical results

For the theoretic-numerical determination of acoustic fields connected with propagating pulses and their spectra the numerical procedure called WJ Code [19] was applied. It was developed recently for the nonlinear propagation equation (24) published by WÓJCIK [18]. A circular transducer, 2 cm in diameter, with an approximate parabolic pressure distribution on its surface (see Fig. 3) was assumed as the wave source. Boundary conditions for a plane transducer were assumed as in the paper [12]. One should
notice that the matching layer on the transducer surface was ignored in numerical calculations. Absorption coefficient for water $\alpha = 28 \cdot 10^{-5} \text{Np/cm MHz}^2$ [2] was applied in calculations.

To determine the critical distance, where distortions caused by nonlinear propagation in water were maximum, a number of measurements were carried out by means of the PVDF hydrophone. Figure 5 shows the pulse shapes and their spectra obtained on

Fig. 5. Pressure pulses and their spectra at various distances $z$ measured with the PVDF hydrophone.
the beam central axis, very near to the transducer at a distance \( z = 8 \text{ mm} \), then at \( z = 132 \text{ mm}, 162 \text{ mm} \) and \( 198 \text{ mm} \). It is evident that the highest number of harmonics arose for distances \( z > 160 \text{ mm} \) (see also Fig. 7). The second and third harmonics which have the highest amplitudes among other harmonics attained maximum values at this distance. So, we assumed that at this distance maximum distortions, caused by nonlinear propagation, were expected.

Fig. 6. The distribution of the pressure in the ultrasonic pulse as a function of the distance from the central beam axis computed for the carrier frequency (top) and for the 5-th harmonic (bottom). Moduli in relative units (left) are shown and phases in radians (right). The distance \( z = 168 \text{ mm} \).

Figure 6 demonstrates, as an example, of the distribution of the pressure as a function of the distance \( r \) from the central beam axis for the distance of \( z = 168 \text{ mm} \) computed for the carrier frequency and for the 5-th harmonic. Taking into account the length of the gold conductor equal to 1.3 mm, which is evaporated on the surface of the perspex block, it was observed that at the distance \( r = 0.65 \text{ mm} \) the amplitude drop equaled 0.5%
and the phase change equaled only 0.35° while for the 5-th overtone the corresponding values were 2.5% and 1.7°, respectively. Therefore, the generated wave was considered to be practically plane.

Also the corresponding distributions of the first 11 harmonics of the nonlinear pulse were computed along the beam central axis (Fig. 7).

Pressure pulses and their spectra computed and measured by PVDF and EM hydrophones for a transmitter voltage of 280 V_{pp}, at a distance of 168 mm are shown in Fig. 8. Their shapes are similar, however not identical, their spectra differ in the number of harmonics. In the case of the PVDF hydrophone the spectral components are higher in relation to those measured by means of the EM hydrophone. An increase in spectral components of the signals, obtained from the PVDF hydrophone, can be explained by its sensitivity S(f) which increases with frequency.

The theoretical sensitivity characteristics of the PVDF hydrophone were shown in 1988 [5] on the basis of the Mason’s [16] equivalent circuit when the full expression describing the hydrophone sensitivity was given in a general case. For the symmetrical acoustical load a more simple expression representing the ratio of the hydrophone output voltage U to the incident pressure p was obtained

\[
\frac{U}{p} = \frac{2ANZ}{R_B - f\left(\frac{R \cdot \text{ctg} x}{2} - \frac{N^2}{x f_m C_0}\right) + 2ZN^2}
\]  

(1)

where \( Z = Z_i/(1 + j Z_i 2 xf_m C_0) \), \( Z_i \) is the electrical impedance across electrical clamps of the hydrophone, \( C_0 \) is static capacitance of the hydrophone, \( x = \pi f/f_m \), \( f \) is the current frequency, \( f_m \) is mechanical resonance frequency of the hydrophone, and \( N \) is the electromechanical transformation ratio. The pressure equals \( p = F/A \), where \( F \) denotes the force occurring in the Mason’s circuit and \( A \) is the active hydrophone surface, \( R = \rho_p c_p A \) denotes the mechanical impedance of the transducer (plastic foil), \( R_B = \rho_w c_w A \) is the mechanical impedance of the load (water).

Inserting into Eq. (1) the values typical for our laboratory made PVDF hydrophone (see [17]) \( C_0 = 0.36 \text{ pF}, N = hC_0 = 0.75 \cdot 10^{-8} \text{ s A/m}, \) where \( h = 2.08 \cdot 10^6 \text{ N/C}, \) \( Z_i = -1/2\pi f C_C, C_C = 130 \text{ pF} \) is the capacitance used across electrical clamps of the hydrophone, \( R = \rho_p c_p A = 0.758 \text{ kg/s}, \rho_p c_p = 3.8 \cdot 10^6 \text{ kg/m}^2 \text{s} \) (for plastic foil), \( A = 19.6 \cdot 10^{-8} \text{ m}^2, R_B = \rho_w c_w A = 0.294 \text{ kg/s}, \rho_w c_w = 1.5 \cdot 10^6 \text{ kg/m}^2 \text{s} \) (for water), a reasonable sensitivity of the hydrophone equal to 17.5 mV/MPa was obtained for the frequency of 3 MHz. However, for example at a higher frequency of 25 MHz the sensitivity calculated from Eq. (1) was higher, being equal to 22.5 mV/MPa as shown in Fig. 9.

The mechanical resonance frequency \( f_m = 43 \text{ MHz} \) corresponded to the plastic foil 25 μm thick. However, the gold layers of electrodes, covering the plastic foil, can decrease considerably the resonance frequency to a value of about 30 MHz [7, 9]. So, the effective frequency band, used in the measurements, is situated on the increasing slope of the resonance curve, below the resonance, and therefore the hydrophone sensitivity increases considerably with frequency.

The PVDF bilaminar membrane hydrophone (Model 804-041) which was used in course of this work for exact measurements was much more sensitive than our laboratory
Fig. 7. The modulus of the first 11 harmonics as a function of the distance $z$ from the transducer. The numbers of harmonics are given at the corresponding curves.
Fig. 8. Pressure pulses and their spectra: computed (top), measured directly with the PVDF (middle) and with the EM hydrophone (bottom) at the distance of z = 168 mm.

Fig. 9. Sensitivity of the PVDF hydrophone calculated from Eq. (1).
made model (compare Table 1 and Fig. 9), however, relative sensitivity characteristics of both the hydrophones were similar as a function of frequency.

Fig. 10. Pressure pulse and its spectrum measured by the PVDF hydrophone at the distance of 168 mm for the transmitter voltage of 280 $V_{pp}$ (top) and 47 $V_{pp}$ (middle) before correction and for the transmitter voltage of 280 $V_{pp}$ after correction (bottom).

Figure 10 shows the pressure pulse and its spectrum measured by means of the PVDF hydrophone at a distance of 168 mm for transmitter voltages of 280 $V_{pp}$ (top) and 47 $V_{pp}$ (middle). Higher harmonics obtained in the second case were much lower. For example, the 5-th overtone was relatively 15 times lower. This case showing a rather quasi-linear pulse propagation in water was called a linear case in the first approximation.
5. Correction of pulses measured by means of the PVDF hydrophone

To eliminate the undesirable effect of sensitivity increase with frequency which introduces distortions of the measured pulse, a correction of the sensitivity characteristic $S(f)$ was carried out. In our calculations only the absolute value $|S(f)|$ was used, since only this information was published by the producer.

Performing the correction, first the sensitivity $|S(f)|$ was approximated by an analytical curve [15]. Next, the correction function $CF_{\text{PVDF}} = 1/|S(f)|$, normalized in respect to the frequency of 2 MHz, was formed. Multiplying the complex spectrum of the pulse measured at a distance of 168 mm (Fig. 10, top) by the function $CF_{\text{PVDF}}$ in the frequency range from 1 MHz to 20 MHz and performing next the inverse Fourier transform, the corrected pressure pulse and the corresponding spectrum were obtained (Fig. 10, bottom).

Comparing the pressures at a distance of 168 mm before and after the correction (Fig. 10 top and bottom) a decrease of the corrected maximum peak-to-peak pressure equal to 27% was found (see also Table 2). The corresponding value for the positive pressure component was even higher and equal to 32%. This rather high correction value was caused by the high rate of the sensitivity increase with frequency. The corrected spectrum showed also a decrease of higher harmonics in respect to the measured one.

Table 2. Results of measurements by means of the PVDF hydrophone.

<table>
<thead>
<tr>
<th>Distance z [mm]</th>
<th>8</th>
<th>132</th>
<th>150</th>
<th>162</th>
<th>168</th>
<th>180</th>
<th>192</th>
<th>198</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmitter voltage $U$ [V&lt;sub&gt;pp&lt;/sub&gt;]</td>
<td>280</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Measured voltage $u$ [mV&lt;sub&gt;pp&lt;/sub&gt;]</td>
<td></td>
<td>27.1</td>
<td>45.6</td>
<td>51.8</td>
<td>54.2</td>
<td>58.9</td>
<td>55.7</td>
<td>52.9</td>
</tr>
<tr>
<td>Pressure* $p = u/S$ [MPa]</td>
<td>0.968</td>
<td>1.63</td>
<td>1.85</td>
<td>1.94</td>
<td>2.10</td>
<td>1.99</td>
<td>1.89</td>
<td>1.90</td>
</tr>
<tr>
<td>Positive peak pressure* [MPa]</td>
<td>0.484</td>
<td>1.11</td>
<td>1.35</td>
<td>1.48</td>
<td>1.63</td>
<td>1.54</td>
<td>1.44</td>
<td>1.48</td>
</tr>
<tr>
<td>Negative peak pressure* [MPa]</td>
<td>0.484</td>
<td>0.52</td>
<td>0.50</td>
<td>0.46</td>
<td>0.47</td>
<td>0.45</td>
<td>0.45</td>
<td>0.42</td>
</tr>
<tr>
<td>Corrected pressure** [MPa]</td>
<td>1.43</td>
<td>1.53</td>
<td>1.55</td>
<td>1.54</td>
<td>1.57</td>
<td>1.48</td>
<td>1.47</td>
<td></td>
</tr>
<tr>
<td>Transmitter voltage $U$ [V]</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>47</td>
</tr>
<tr>
<td>Measured voltage $u$ [mV&lt;sub&gt;pp&lt;/sub&gt;]</td>
<td></td>
<td>4.60</td>
<td>6.04</td>
<td>5.95</td>
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<td>5.87</td>
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<td>Pressure* $p = u/S$ [MPa]</td>
<td>0.164</td>
<td>0.216</td>
<td>0.212</td>
<td>0.214</td>
<td>0.210</td>
<td>0.208</td>
<td>0.206</td>
<td>0.205</td>
</tr>
</tbody>
</table>

* Calculated for the sensitivity of $S = 28$ mV/MPa corresponding to the frequency of 2 MHz.
** After correction of the sensitivity characteristic of the PVDF hydrophone.

Figure 11 presents the pressures measured on the beam axis by means of the PVDF hydrophone at the high (280 V<sub>pp</sub>) voltage as a function of distance, before and after the correction. In all the cases the complex spectrum of the measured pulse was taken into account. At low transmitter voltage (47 V<sub>pp</sub>) the correction was not necessary, since the pulses had a narrow band spectrum with a practically constant sensitivity.
Fig. 11. Results of pressure measurements at different distances before (B) and after (C) correction of the frequency characteristic of the PVDF hydrophone at the high (280 V\text{pp}) transmitter voltage.

6. Correction of pulses measured by means of the EM hydrophone

Because the sensitivity of the EM hydrophone was many times lower than that of the PVDF one, an additional 28 dB amplifier was used to obtain electrical signals of comparable values from both the hydrophones. The amplifier itself caused some frequency distortions of the measured spectrum as shown in Fig. 12 together with small distortions produced by the nonideal frequency characteristic of the EM hydrophone, as presented in Fig. 2. The applied correction procedure was the same as described above; however, complex frequency characteristics of the amplifier and of the EM hydrophone were used in this case. Figure 13 presents the pulse and its spectrum measured by the EM hydrophone at a transmitter voltage of 280 V\text{pp} before (top) and after correction (middle). The corrected maximum peak-to-peak pulse pressure at a distance of 168 mm was about 3% higher (see Table 3). At all the distances the complex spectra of the measured pulses were taken into account though their changes were very small.

However, when using a perfect amplifier with a constant frequency response characteristic of up to 20 MHz the corrected pulse pressure measured by the EM hydrophone should be increased by 0.7% only. The very low value of the correction is caused by the almost constant sensitivity of the EM hydrophone as a function of frequency up to 20 MHz.

The pulse pressure measured by means of the EM hydrophone is given in [Pa] by the formula [8]

\[
p = \frac{e(p_c + p_w c_w)}{2Bl},
\]  

(2)
Fig. 12. Modulus $A$ and phase $\phi$ of an additional amplifier as a function of frequency. Measurements — dotted lines, analytical approximating curves — solid lines.

Table 3. Results of measurements by means of the EM hydrophone.

<table>
<thead>
<tr>
<th>Distance $z$ [mm]</th>
<th>132</th>
<th>150</th>
<th>162</th>
<th>168</th>
<th>180</th>
<th>192</th>
<th>198</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transmitter voltage $U$ [V_{pp}]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>280</td>
<td></td>
</tr>
<tr>
<td>Measured voltage $u$ [mV_{pp}]</td>
<td>3.39</td>
<td>3.49</td>
<td>3.31</td>
<td>3.27</td>
<td>3.07</td>
<td>3.19</td>
<td>3.03</td>
</tr>
<tr>
<td>Corrected voltage*** $u$ [mV_{pp}]</td>
<td>3.42</td>
<td>3.51</td>
<td>3.38</td>
<td>3.36</td>
<td>3.15</td>
<td>3.30</td>
<td>3.14</td>
</tr>
<tr>
<td>Pressure $p^{****}$ [MPa_{pp}]</td>
<td>1.37</td>
<td>1.41</td>
<td>1.36</td>
<td>1.35</td>
<td>1.27</td>
<td>1.33</td>
<td>1.26</td>
</tr>
<tr>
<td>Transmitter voltage $U$ [V_{pp}]</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>47</td>
<td></td>
</tr>
<tr>
<td>Measured voltage $u$ [mV_{pp}]</td>
<td>0.43</td>
<td>0.44</td>
<td>0.40</td>
<td>0.43</td>
<td>0.41</td>
<td>0.39</td>
<td>0.36</td>
</tr>
<tr>
<td>Corrected voltage*** $u$ [mV_{pp}]</td>
<td>0.44</td>
<td>0.44</td>
<td>0.41</td>
<td>0.43</td>
<td>0.43</td>
<td>0.40</td>
<td>0.38</td>
</tr>
<tr>
<td>Pressure $p^{****}$ [MPa_{pp}]</td>
<td>0.215</td>
<td>0.215</td>
<td>0.20</td>
<td>0.21</td>
<td>0.195</td>
<td>0.19</td>
<td>0.18</td>
</tr>
</tbody>
</table>

*** After correction of frequency characteristics of the amplifier and of the EM hydrophone.

**** Calculated from Eq. (1).
where \( e \) is the voltage measured across the terminals of the hydrophone in [V], \( \rho_{pc} c_p \) and \( \rho_{wc} c_w \) are acoustic impedances of perspex and water in [kg/m²s], \( B \) is magnetic induction in [T], and \( l \) is length of the conductor in [m].

Fig. 13. The pressure pulse and its spectrum measured with the EM hydrophone at a distance of \( z = 168 \text{ mm} \) for the transmitter voltage of 280 V\(_{pp}\) before (top), after correction (middle) and for the transmitter voltage of 47 V\(_{pp}\) (bottom).

7. Comparison of corrected PVDF and EM measurements

Applying the EM hydrophone under the same conditions as those for the PVDF hydrophone (Fig.1) at a distance of \( z = 168 \text{ mm} \), the value \( e = 3.27 \text{ mV}_{pp}/25 \) was
obtained for the transmitter voltage of 280\text{V}_{\text{pp}} (\text{Fig. 13, top})]. The number 25 in the denominator was equal to the additional amplification used in this case, the other values being: \( B = 0.19 \text{T}, \ l = 0.0013 \text{m}, \ \rho_{\text{p}} c_{\text{p}} = 3.2 \cdot 10^6 \text{kg/m}^2\text{s} \) and \( \rho_{\text{w}} c_{\text{w}} = 1.5 \cdot 10^6 \text{kg/m}^2\text{s} \).

Hence, the pressure determined from Eq. (2) was
\[
p = 3.27 \text{mV}_{\text{pp}} \cdot (25 \cdot 2 \cdot 0.19 \text{[T]} \cdot 0.0013 \text{[m]})^{-1} \cdot 4.7 \text{Mrayl} = 1.24 \text{MPa}_{\text{pp}}.
\]

So, the pressures measured by the PVDF and EM hydrophones at the distance of \( z = 168 \text{mm} \) were \( p_{\text{PVDF}} \) = 56.9 mV_{pp} / 28 mV/MPa = 2.0 MPa_{pp} (see Fig. 10, top) and \( p_{\text{EM}} \) = 1.24 MPa_{pp}, respectively. Decreasing, according to the correction data, the output of the PVDF hydrophone by 27% and increasing the output of the EM hydrophone by 3% a difference of 0.19 MPa_{pp} corresponding to 12% of the maximum pressure value was obtained (see Table 4). This difference may be caused by the fact that our considerations were limited to the highest calibration frequency of the PVDF hydrophone equal to 20 MHz, ignoring in this way spectral components beyond this frequency. Also the uncertainty in the sensitivity calibration of the PVDF hydrophone should be taken into account. It equaled from \( \pm 7\% \) to \( \pm 12\% \) according to the certificate of NPL (Teddington, England).

**Table 4. Difference \( \Delta p = p_{\text{PVDF}} - p_{\text{EM}} \) between corrected pressures measured by means of the PVDF and EM hydrophones.**

<table>
<thead>
<tr>
<th>Distance ( z ) [mm]</th>
<th>132</th>
<th>150</th>
<th>162</th>
<th>168</th>
<th>180</th>
<th>192</th>
<th>198</th>
</tr>
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<td>280</td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pressures ( p_{\text{PVDF}} ) [MPa_{pp}]</td>
<td>1.43</td>
<td>1.53</td>
<td>1.55</td>
<td>1.54</td>
<td>1.57</td>
<td>1.48</td>
<td>1.47</td>
</tr>
<tr>
<td>Pressures ( p_{\text{EM}} ) [MPa_{pp}]</td>
<td>1.37</td>
<td>1.41</td>
<td>1.36</td>
<td>1.35</td>
<td>1.27</td>
<td>1.33</td>
<td>1.26</td>
</tr>
<tr>
<td>( \Delta p = p_{\text{PVDF}} - p_{\text{EM}} ) [MPa_{pp}]</td>
<td>0.06</td>
<td>0.12</td>
<td>0.19</td>
<td>0.19</td>
<td>0.30</td>
<td>0.15</td>
<td>0.21</td>
</tr>
<tr>
<td>Mean (( \sum \Delta p ))/7 [MPa_{pp}]</td>
<td>0.174 ( \leftrightarrow 12% )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Transmitter voltage ( U ) [V_{pp}]</td>
<td>47</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pressures ( p_{\text{PVDF}} ) [MPa_{pp}]</td>
<td>0.216</td>
<td>0.216</td>
<td>0.214</td>
<td>0.209</td>
<td>0.208</td>
<td>0.206</td>
<td>0.205</td>
</tr>
<tr>
<td>Pressures ( p_{\text{EM}} ) [MPa_{pp}]</td>
<td>0.216</td>
<td>0.216</td>
<td>0.200</td>
<td>0.210</td>
<td>0.195</td>
<td>0.191</td>
<td>0.183</td>
</tr>
<tr>
<td>( \Delta p = p_{\text{PVDF}} - p_{\text{EM}} ) [MPa_{pp}]</td>
<td>0.000</td>
<td>-0.004</td>
<td>0.014</td>
<td>-0.001</td>
<td>0.013</td>
<td>0.015</td>
<td>0.022</td>
</tr>
<tr>
<td>Mean (( \sum \Delta p ))/7 [MPa_{pp}]</td>
<td>0.0084 ( \leftrightarrow 4% )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

To eliminate the effect of frequency characteristics of both the hydrophones additional measurements were performed at a lower voltage of 47 V_{pp}. Then the spectrum of the pulse corresponded to a narrow frequency band around a carrier frequency of 2 MHz. The pressure measured by the PVDF hydrophone at the distance of \( z = 168 \text{mm} \) was equal to \( p_{\text{PVDF}} \) = 5.89 mV_{pp} / (28 mV/MPa) = 0.21 MPa_{pp}. Figure 13 (bottom) shows the same pulse and its spectrum measured by means of the EM hydrophone. One can observe here a high level of interfering signals arising from other electrical equipment situated near-by. The voltage value obtained in this case equaled 0.55 mV_{pp}, hence the same pressure, namely \( p_{\text{EM}} \) = 0.21 MPa_{pp} was calculated from Eq. (2). Almost the same pressure was measured with PVDF and EM hydrophones at distances of 132, 150, 162, 168, 180, 192 and 198 mm (see Table 4).
From Eq. (2) it was possible to determine directly the sensitivity of the EM hydrophone in the case of linear propagation, namely:

\[
\frac{e}{p} = 2B (\rho_p c_p + \rho_w c_w)^{-1} = 2 \cdot 0.19 \llbracket T \rrbracket 1.3 \cdot 10^{-3} \llbracket m \rrbracket \{ (3.2 + 1.5) \cdot 10^6 \llbracket kg/m^2s \rrbracket \}^{-1} = 0.10 \text{mV/MPa.}
\]  

(3)

Hence, one can conclude that the sensitivity of the EM hydrophone, equal to 0.10 mV/MPa, was 280 times lower than that of the PVDF hydrophone equal to 28 mV/MPa at a frequency of 2 MHz.

![Graph](image)

Fig. 14. Pressure distribution along the beam axis computed (N) and measured by the PVDF hydrophone before (B) and after correction of its frequency characteristic (C), also measured by the EM hydrophone, after correction. Measurements were performed at high (280 V_{pp}) and low (47 V_{pp}) transmitter voltages generating nonlinear and linear pressure levels, respectively.

Figure 14 shows a comparison of the measurement results obtained by means of the PVDF and the EM hydrophones after corrections. The average difference of the two methods was equal to 12% in the nonlinear case (for the transmitter voltage of 280 V_{pp}) and to 4% in the linear case (for the transmitter voltage of 47 V_{pp}) (see Table 4). Hence, one can conclude that our EM hydrophone measured the pressures correctly.

The agreement between the computed and measured pressure distributions along the beam axis is fairly good with the exception of the distance z = 30 mm - 90 mm, which corresponded to the region of interference between the central axial wave and the edge wave [1]. This effect was caused by the \( 1/4 \) matching layer covering the radiating transducer that changed the edge wave. To verify this supposition measurements were performed by means of the PVDF hydrophone along the beam axis with an ultrasonic transducer radiating directly a pressure pulse without a matching layer. Figure 15 shows the obtained computed and measured distributions of the pulse pressure at the distance
of \( z = 0 - 100 \text{mm} \). The measured pressures are lower than in Fig. 14 due to the lack of the matching layer; however, the shapes of the measured and computed curves are almost identical. In this way the agreement between the computed and measured curves showing the pulse pressure distribution along the whole beam axis was confirmed.

![Graph](image)

Fig. 15. Pressure distribution along the beam axis computed as in Fig. 14 (dotted line) and measured (solid line with points) by means of the PVDF hydrophone at high \( 280 \text{ V}_{pp} \) transmitter voltage in the case of a transducer without a matching layer.

However, it arises a question if the hydrophones used in measurements were linear, otherwise one could expect serious distortions of the obtained data. The linearity of the PVDF hydrophone was shown up to pressures of 2.3 MPa by means of a newly developed multilayer method [10]. The EM hydrophone is linear in its principle. However, it was also necessary to demonstrate that the acoustic impedances in the formula (2) are practically not dependent on the nonlinearity of the measured pulses. This was already shown by the formula (7) in the paper [6] devoted to the EM hydrophone used for shock wave measurements, and derived in a previous publication [11].

8. Conclusions

Nonlinear propagation effects of short finite amplitude pulses radiated by a plane transducer with pressures of about 1.5 MPa\(_{pp}\) and a carrier frequency of 2 MHz were computed by the numerical code recently developed by the last-named author and were measured in a water tank. Measurements by means of a calibrated PVDF hydrophone
showed a higher number of spectral components (harmonics) than those by an electromagnetic (EM) hydrophone due to sensitivity characteristics, increasing in the PVDF hydrophone and decreasing in the EM hydrophone as a function of frequency, as shown by analysis.

Measurements were performed at several distances from the wave source at high and low transmitter voltages corresponding to nonlinear and linear propagation. Corrections of the distortions caused by frequency characteristics of both the hydrophones demonstrated that in the investigated PVDF hydrophone, the maximum measured peak-to-peak pulse pressure was by 27% too high while in the EM hydrophone by 0.7% too low. However, when using an additional amplifier with the EM hydrophone its frequency characteristic should also be taken into account. Then the corrected pressure was by 3% too low in the investigated case.

Sensitivities of the PVDF and the EM hydrophones at a frequency of 2 MHz were equal to 28 mV/MPa and 0.10 mV/MPa, respectively. The calibration of the EM hydrophone was performed by two simple electrical and magnetic measurements. In this way it is also possible to calibrate PVDF hydrophones by comparison.

A good agreement was obtained between the computed results and those measured by two different methods, demonstrating the pulse pressure distribution along the whole beam axis. In the nonlinear case, the average difference of obtained results equaled 12% while in the linear case, 4% only.

It was also shown that the $\lambda/4$ matching layer covering the transducer surface influenced the edge wave produced by the transducer.

The numerical WJ code made it possible to simulate exactly nonlinear propagation effects independently of the distance from the acoustic source.

Acknowledgements

The first author of this paper learned with sorrow of the death of Professor Z. JAGODZINSKI, a wondrous man, who was his first chief at the MORŚ Comp. in Gdynia, teaching him radio-communication sea service in 1948. Then, he became his close friend cooperating in many fields of ultrasonic problems.

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References


[19] J. Wójcik, see the section *Basis of numerical procedures in the reference 12.*