DISLOCA TION CONTRIBUTION IN THE ACOUSTOELASTIC EFFECT

J. DEPUTAT

Institute of Fundamental Technological Research of the Polish Academy of Sciences
(00-049 Warszawa, ul. Świętokrzyska 21)

1. Introduction

The acoustoelastic effect consists in the relationship between elastic deformation and velocity of acoustic waves in the medium. Properties of this phenomenon are most frequently observed in the course of measurements of changes of travel time of very small amplitude ultrasonic waves propagating in solids subjected to loading with external forces. A theoretical model is approximated in such experiments by superimposing infinitely small variable displacements on finite elastic deformation. Velocity increments of ultrasonic waves due to stress are usually small and high time resolution apparatus is necessary for their measurement. Velocity changes due to stress are proportional to stress and in a given medium they depend on the direction of wave propagation and orientation of particle vibration in the wave in relation to direction of stress. A 10 MPa increase of tensile stress in steel decreases velocity of longitudinal waves propagating in the direction of stress by about 0.75 m/s; longitudinal waves propagating perpendicular to stress by about 0.1 m/s; transverse waves propagating in the direction of stress by about 0.07 m/s; transverse waves propagating perpendicular to stress and polarized in the direction of stress by about 0.4 m/s, while those polarized perpendicular to the direction of stress by about 0.1 m/s. These changes depend on type of material, and for example in the case of aluminium and its alloys they are approximately twice as big and in plastics many times greater than in steel. The linear theory of elasticity does not lead to a relationship between elastic deformation and the velocity of acoustic waves. According to the linear theory of elasticity velocities of acoustic waves in an isotropic unbounded elastic medium are expressed by:

\[ V_L = \sqrt{\frac{E(1-v)}{\rho(1+v)(1-2v)}} \]
\[ V_T = \sqrt{\frac{G}{\rho}} \]

(1)
where: $V_L$ and $V_T$ are velocities of longitudinal and transverse waves, respectively; $E$ and $G$ are longitudinal and shear moduli of elasticity (second-order elasticity constants), respectively; $\nu$ - Poisson's ratio; $\rho$ - mass density. A relationship between velocity of acoustic waves and stress is reached when the non-linear stress-deformation dependence is included. Initial form of expressions relating velocity of elastic waves with low amplitude, and stress for an isotropic medium were given by Hughes and Kelly [1] in 1953. Their paper initiated wide theoretical and experimental research on the acoustoelastic effect, as well as trials of implementing the stress-velocity relation in measurements of residual stresses in materials and structure elements. An extensive review of theoretical and experimental work concerning the acoustoelastic effect can be found in monographs and review papers [2–5]. The velocity, $V$, of waves in a body elastically deformed by uniaxial stress, $\sigma$, is expressed by second-order elasticity constants, $\lambda$ and $\mu$ (Lame's constants at $\sigma = 0$), mass density in neutral state, $\rho_0$, and third-order elasticity constants, $k$, $l$ and $m$. Expressions achieved in paper [1] for longitudinal and transverse waves propagating in the direction of stress have the following form:

$$\rho_0 V_{111}^2 = \lambda + 2\mu - \frac{\sigma}{3K_0} \left[ 2l + \lambda + \frac{\lambda + \mu}{\mu} (4m + 4\lambda + 10\mu) \right]$$

(2)

$$\rho_0 V_{212}^2 = \mu - \frac{\sigma}{3K_0} \left[ m + \frac{\lambda n}{4\mu} + 4\lambda + 4\mu \right].$$

(3)

Succeeding indexes of $V$ denote the direction of wave propagation, direction of particle's vibrations and direction of stress. Velocity changes due to stress described by formulae (2) and (3) are related with non-linear properties of elastic interaction of particles in the medium. Experiments indicate practical linearity of the $\sigma$-$v$ dependence and in strain measurements the expression in used:

$$\frac{v - v_0}{v_0} = \frac{t_0 - t}{t} = \beta \cdot \sigma$$

(4)

where: $t_0$ and $t$ are travel times of waves over a constant path in material in initial and deformed state, respectively; $\beta$ is the acoustoelastic constant of the material for a given configuration between directions of stress, propagation and polarization of waves in hitherto performed investigations. A symmetry of the effect is observed when the sign of stress is changed. Replacing tension with compression causes a change of sign of velocity increment only. Value changes of the acoustoelastic constant have not been noted when the sign of stress was changed. A frequency dependence of the acoustoelastic constant was not stated. No significant changes of the acoustoelastic constant have been stated for different chemical composition of steel, within limits permissible by standards, and the same $\beta$ value is accepted for a given grade of steel [6]. Research on the influence of metalurgical phase transformation on the value of the $\beta$ constant are performed [7].

The dislocation theory indicates the possibility of occurrence of additional
velocity increments due to stress, when dislocations, vibrating in the stress field of propagating acoustic waves, can be found in the crystal. Such dislocations are called active dislocations to tell them from dislocations pinned down on the whole length by point defects and dislocation network points. Active dislocations are introduced into the material through plastic deformation. A hypothesis was made that in plastically deformed construction materials dislocation changes of velocity can occur beside changes due to elastic non-linearity of the crystal lattice. Additional velocity changes due to stress should be taken into account in stress investigations based on velocity measurements of ultrasonic waves.

This paper is dedicated to initial research on the acoustoelastic effect in plastically deformed steel sample. Values of acoustoelastic constants were determined in 24H2MF and 45 steel in initial state and after plastic deformation. The occurrence of an additional source of velocity changes due to stress in plastically deformed samples was stated. Properties of additional wave velocity increments were investigated and the dislocation character of this phenomenon was stated. Results were evaluated on grounds of the theory of dislocation modulus defect (changes of the modulus of elasticity) based on the model of dislocations in the form of a vibrating string [8].

2. Experiments

In measurements with waves propagating in the direction of stress, rod-shaped samples with circular crosssection diameter of 16 mm and length of 180 mm were used. Sample ends at the length of 18 mm were threaded in order to mount the sample in holders of a machine for tensile tests. Piezoelectric transducers, which were sources of ultrasonic waves, were applied to the flat surface of the sample's end face. Measurements with waves propagating perpendicular to the direction of stress were made on rod-shaped samples with rectangular section in the measurement part. Piezoelectric transducers were applied to the flat side surface of samples.

Changes of the travel time of pulses of ultrasonic waves between chosen bottom echos were measured during loading with a tension and compression, when samples were in a state after stress relieving annealing (573 K, 24 h) and after plastic deformation. Plastic deformation was introduced by stretching samples. Measurements were carried out at constant room temperature with a meter of travel time of pulses of ultrasonic waves. The meter provided ±1 ns resolution. From measured values of travel time a part of time change resulting from wave’s path increment due to elastic deformation of sample was subtracted:

\[
\left( \frac{t_0 - t}{t} \right)_a = \left( \frac{t_0 - t}{t} \right)_{\text{measured}} \cdot \frac{l_0 - l}{l} \tag{5}
\]

where: \( l_0 - l \) is wave's path increment due to elastic deformation. In the case of
measurements with waves propagation along the sample, the stress distribution on the axis of loaded sample was included by introducing the shape coefficient, as it is described in paper [9].

3. Results of measurements

Fig. 1 presents examples of velocity changes of longitudinal waves with frequency of 4 MHz versus time after plastic deformation of a 24 H2MF steel sample. Measurements were done at room temperature. After deformation a recovery of wave velocity is observed. Recovery is practically full after 48 hours.

Fig. 2 presents relative changes of travel time of longitudinal ultrasonic waves with frequency of 4 MHz and 10 MHz, propagating in the direction of stress in terms of stress value in the 24 H2MF steel sample before plastic deformation $\varepsilon = 0$ and after plastic deformation $\varepsilon = 3\%$. Data is arranged along straight segments. The repeatability of results is better than 1%. Tangents of inclinations of these straight lines are acoustoeelastic coefficients for individual cases.

The acoustoeelastic coefficient for longitudinal waves in a sample before plastic deformation is equal to $\beta = -1.77 \times 10^{-5}$ MPa$^{-1}$ and it has the same value for tension and compression of the sample. Values measured for waves with frequencies of 4 MHz and 10 MHz lie on the same straight line. Due to a 3% plastic deformation the value of the acoustoeelastic coefficient changes become depending on the direction of loading. Also a frequency dependence of the coefficient is observed. Values of the coefficient determined during compression are higher than during tension. $\beta$ values achieved during compression are higher for 10 MHz frequency than for 4 MHz. During tension the $\beta$ value for waves with frequency of 10 MHz is smaller than for waves with frequency of 4 MHz.
The acoustoelectric constant varies with plastic deformation. In plastically deformed materials, the value of velocity changes and the sign of stress is observed. A dependence on the stress is observed under the same stress when the degree of plastic deformation increases. A frequency dependence of the acoustoelectric constant is obtained at a plastic deformation of a plastically deformed sample. A dependence on the properties exhibiting the sample in a deformed state. Such properties of the phenomenon indicate the occurrence of a second source of velocity changes due to stress. The source is associated with elastic nonlinearity in the crystal lattice. This additional source should introduce velocity changes with the same sign independent of the direction of loading (velocity decreases during compression, as well as during tension). Velocity changes introduced by an additional source should depend on the degree of plastic deformation and exhibit dispersion. Such properties are demonstrated by the velocity changes of ultrasonic waves resulting from the so-called dislocation defect of the modulus consisting in a decrease of the modulus of elasticity due to the presence of active dislocations.

In crystals containing active dislocations, the propagation of ultrasonic waves is observed in contrast to the velocity in the same crystals with pinned dislocations. The value of the modulus of elasticity in a crystal containing active dislocations is lower than that of ordinary crystals. The introduction of active dislocations (the crystal contains plastic deformation) leads to a decrease of velocity of ultrasonic waves related to the motion of existing dislocations and a decrease of the modulus of elasticity due to the introduction of active dislocations has been known in pure monoclines. Under the influence of stress, the elastic modulus of the material is lower than in the case when the crystal lattice does not contain dislocations.
Results of measurements of the acoustoelastic constant for longitudinal waves $\beta^L$ in samples with various values of plastic deformation have been gathered in Fig. 3. Differences in the value of $\beta$ determined during compression and tension initially grow with an increase of plastic deformation, while for a deformation exceeding approximately 1.5% $\beta$ increments are small. Differences of $\beta$ value determined for 4 MHz and 10 MHz are measurable for plastic deformations exceeding 1.5%.

Figs. 4 and 5 present results of measurements achieved for samples of steel 45. A relationship between the coefficient for transverse waves propagating perpen-

---

**Fig. 4**

- 45 waves $L \parallel 6^\circ$
- determined during compression
- determined during tension

---

**Fig. 5**

- 45 waves $T \parallel 6^\circ$
- determined during compression
- determined during tension
diccular to the direction of stress and polarized in the direction of stress, and value of plastic deformation is shown in Fig. 4. Fig. 5 presents results of investigations of this coefficient for longitudinal waves propagating in the direction of stress. The character of the dependence resembles that for 24 H2MF steel.

Values of the acoustoelastic coefficient for longitudinal waves propagating along the direction of stress in a sample of 24 H2MF steel, which after 3% plastic deformation was annealed for 24 hours at the temperature of 573 K, determined during compression and tension, are equal and within the measuring error equal to value of $\beta$ measured for this sample in initial state. A frequency dependence of $\beta$ was not observed.

The acoustoelastic effect in well annealed samples is fully symmetrical in relation to the sign of stress and does not display a frequency dependence. In plastically deformed samples a dependence between the value of velocity changes and sign of stress is observed, as well as an increase of values of changes under the same stress when the degree of plastic deformation is increased, and a frequency dependence of the acoustoelastic coefficient is stated. Annealing of a plastically deformed sample causes the recovery of properties exhibited by the sample in initial state. Such properties of the phenomenon indicate the occurrence of a second source of velocity changes due to stress, besides the source described with elastic nonlinearity of the crystal lattice. This additional source should introduce velocity changes with the same sign independently of the direction of loading (velocity decreases during compression, as well as during tension). Velocity changes introduced by an additional source should be dependent on the degree of plastic deformation and exhibit dispersion. Such properties are demonstrated by velocity changes of ultrasonic waves resulting from the so-called dislocation defect of the modulus consisting in a decrease of the value of the modulus of elasticity due to the presence of active dislocations.

4. Dislocation changes of velocity

In crystals containing active dislocations a lower velocity of ultrasonic waves is observed in comparison to the velocity in the same crystals with pinned dislocations. The value of the modulus of elasticity in crystals containing active dislocations is also lower than in a case of pinned dislocation loops. Thus the introduction of active dislocations into the crystal (e.g. through plastic deformation) leads to diminishing of the modulus of elasticity (modulus defect) and a decrease of velocity of ultrasonic waves related to the modulus. The phenomenon of a decrease of the modulus of elasticity due to the introduction of active dislocations has been known in pure monocrystals. Under the same stress elastic deformation is greater in crystalline media containing active dislocations than in the case when the crystal lattice does not contain dislocations, or when the motion of existing dislocations is
impossible. Total elastic deformation consists of displacement described by the stress-deformation dependence, \( \varepsilon_s \), and dislocation deformation, \( \varepsilon_d \):
\[
\varepsilon = \varepsilon_s + \varepsilon_d
\]  
(6)

The first component of deformation is expressed by:
\[
\varepsilon_{ijkl} = C_{ijkl} \sigma_{ij}
\]  
(7)

where: \( C \) — elasticity constants and \( \sigma \) — stress. Deformation due to a dislocation loop with length \( l \) in the volume unit is expressed by a product of average displacement \( \xi \), loop length \( l \) and modulus of Burgers vector \( b \). The average displacement of a dislocation \( \xi \) is determined by:
\[
\xi = \frac{1}{l} \int_0^l \xi(y)dy
\]  
(8)

where: \( y \) is the coordinate along the line of undeformed dislocation (Fig. 6). If \( A \) is the total length of active dislocation loops in a unit volume, then the dislocation deformation is:
\[
\varepsilon_d = \frac{AB}{l} \int_0^l \xi(y)dy
\]  
(9)

A decrease of the modulus of elasticity due to the presence of dislocations manifests itself by a diminishing of velocity of ultrasonic waves, according to the dependence (1):
\[
\frac{\Delta G}{G} = 2\frac{\Delta V}{V}
\]  
(10)

The decrease of the modulus of elasticity is accompanied by an increase of dislocation internal friction. Both these phenomena are described by theories of dislocation internal friction. The most fully verified Granato-Lücke theory [8], based on the string model of dislocation loop, is developed from the equation of dislocation loop's motion:
\[
A \frac{\partial^2 \xi}{\partial t^2} + B \frac{\partial \xi}{\partial t} - C \frac{\partial^2 \xi}{\partial y^2} = b \cdot \sigma_0 \sin \omega t
\]  
(11)

where \( \xi \) is the coordinate in the direction of displacement in the slip plane, \( y \) — coordinate in the direction of line length, \( A = \pi gb^2 \) — mass of a unit of line length, \( B \) — constant of dislocation motion damping, \( C = 2Gb/\pi(1-\nu) \) — stress of a bent dislocation line, \( \rho \) — mass density, \( G \) — elastic modulus, \( \nu \) — Poisson’s ratio, \( b \) — modulus of Burgers vector of dislocations \( \sigma_0 \) — amplitude of applied stress, \( \omega \) — circular frequency of changes of applied stress.
The solution of the equation leads to an expression for a relative change of wave velocity due to the presence of a dislocation loop with length $l$:

$$
\frac{\Delta V}{V}(l) = \frac{V - V_0(l)}{V_0} = \frac{4 \cdot G \cdot b^2 \Omega}{\pi^2 A} \frac{\omega_0^2 - \omega^2}{(\omega_0^2 - \omega^2) + (\omega d)^2}
$$

where: $V_0$ — wave velocity in a crystal without active dislocations; $V$ — velocity in a crystal with dislocations; $A$ — length of line of dislocations in a unit-volume; $\omega_0 = \pi / \sqrt{C / A}$ — resonance frequency of the loop; $d = B / A$, $\Omega$ — orientation factor including the dependence between stress in the direction of wave propagation and stresses in individual slip systems, $\omega$ — frequency of ultrasonic wave driving vibrations of the loop. If the distribution of the length of dislocation loop is taken into consideration, then the expression for velocity change will be as follows:

$$
\frac{\Delta V}{V}(\omega) = \int_0^\infty \frac{\Delta V}{V}(l) \cdot l \cdot N(l) dl
$$

where $N(l)dl$ is the number of loops in a unit of volume with length contained in an interval from $l$ to $l + dl$, and integral of $lN(l)$ substituted dislocation density $A$ in equation (12). For a random distribution of pinning points we reach:

$$
N(l)dl = \frac{A}{L_c^2} e^{-l/L_c} dl.
$$

Due to applied stress loops will bend into arcs (see Fig. 6) and the average free length of the loop will change from $L_c$ to $L_c + \delta L_c$. The influence of stress on changes of damping of ultrasonic waves was analyzed in paper [11]. However, the phenomenon of dislocation changes of velocity due to stress has not been hitherto investigated. If
the increment of the length of a loop $\delta L_c$ will be small in relation to $L_c$, then velocity changes of ultrasonic waves due to stress applied to the sample will be noted as:

$$\delta \left[ \frac{AV}{V(\omega)} \right] = \frac{\delta}{\delta L_c} \left[ \frac{\infty}{0} \frac{AV}{V(l)} \cdot l \cdot N(l) \cdot dl \right] \delta L_c$$

$$= \frac{A}{L_c^3} \delta L_c \left[ \frac{\infty}{0} \frac{AV}{V(l)} \cdot l \left( \frac{l}{L_c} - 2 \right) e^{ilL_c} dl. \right.$$  (15)

Inserting expression (12) in (15) and including the relation between the length increase of the loop and changes of resonance frequency $dl = \pi(\omega_0)^{-1}(C/A)^{-2}$ we reach an expression for measured value of dislocation velocity increment:

$$\delta \left[ \frac{AV}{V(\omega)} \right] = \frac{4 \cdot G \cdot \Omega \cdot b^2 \Lambda^2}{\pi^2 L_c^3 A} \left[ \frac{K}{L} - \omega^2 \right] \cdot \left( \frac{l}{L_c} - 2 \right) e^{-ilL_c} \int \left( \frac{K}{L} - \omega^2 \right)^2 + (\omega d)^2 \cdot dl \right.$$  (16)

where: $K = \pi^2 C/A$.

The length of the loop $L_c$ (initial state), occurring in expression (14), corresponds to resonance frequency $\omega_0$, while the length of the loop $L$ (state after stress applied) corresponds to lower resonance frequency $\omega'$. (14)

Expression (16) indicates that the size of velocity changes of ultrasonic waves related to the presence of active dislocations due to the application of stress depends on the density of dislocations, free length of the loop at the state before stress is applied, length increase of the loop due to applied stress, and on the direction of propagation of waves in relation to direction of grains orientation in the sample.

### 5. Discussion of results

The change of wave velocity observed in plastically deformed steel samples under elastic load consists of increments related to the acoustoelastic effect of the lattice of tested material (expression (2) and (3)) and increments resulting from length change of dislocation loops (expression (17)):

$$\frac{(V - V_0)}{V_0} \text{measured} = \left( \frac{(V - V_0)}{V_0} \right) \text{lattice} + \left( \frac{(V - V_0)}{V_0} \right) \text{dislocation}.$$  (17)

When the direction of vibrations of the medium's particles during the propagation of a wave is consistent with the direction of stress, then during compression the wave velocity is higher than in the initial state and during tension velocity decreases. The sign of the first component of measured velocity changes depends on the direction of applied stress. Elastic tension, as well as compression of a sample containing active dislocations leads to an increment of loop length, and hence to
a velocity decrease. The sign of the second component is independent of the sign of applied stress. The dislocation velocity change increases the acoustoelastic velocity diminishing during stretching and decreases the velocity increment during compres-
sion.

The density of dislocation grows during plastic deformation. Research based on the etched pits count indicate a monotonic increase of the density of dislocations accompanying an increase of plastic deformation [12]. The average free length of loops changes simultaneously with an increase of density of dislocations. However, this change can be in more complex relation with plastic deformation, because the density and distribution of pinning points changes with time due to the diffusion of point defects from the bulk to dislocation lines and diffusion along dislocation lines. In the case of higher values of plastic deformation the motion of dislocations in some slip systems can be hindered (pile-ups of dislocations) and further deformation causes crystalline rotation, so not blocked slip systems are positioned advantageously with respect to the direction of deformation. Surface waves were applied in investigations of the dependence of dislocation velocity changes on plastic deformation in iron [13]. Monotonic velocity changes with regard to deformation have been stated for waves travelling in the direction of the deformation. In the case of waves propagating perpendicular to the direction of deformation the dislocation velocity loss initially increases up to 70% deformation, achieves a maximum and decreases during further deformation. This type of dislocation velocity change suggests that the influence of density of dislocations predominates the effect of the average free length decrease of the loop and an increase of dislocation velocity losses due to applied stress should be expected for waves travelling in the direction of plastic deformation (expression (16)). As to other directions of waves with respect to the direction of displacement a different dependence is possible at high values of plastic deformation.

Also the frequency dependence of the phenomenon indicates a dislocation character of additional velocity changes during loading of plastically deformed steel samples. For fixed parameters of the displacement lattice the value of the dislocation velocity change initially increases with an increase of frequency, \( \omega \), of waves passing through the material, achieves a maximum and decreases for a further frequency increase. The observed increase of velocity change due to applied stress indicates that applied frequencies correspond to values on the left side of the minimum of dislocation velocity changes. Changes of wave velocity due to stress increments do not depend on frequency in materials without active dislocations.

6. Conclusions

Changes of wave velocity due to stress in steels with dislocations moving in a field of ultrasonic waves are a sum of changes resulting from inelastic behaviour of an ideal crystal lattice and changes introduced by dislocations.
The lattice part of velocity changes characterizes a given lattice. Its value at
given stress is independent of frequency and wave amplitude. An increase of velocity
is caused by compressive stress in the direction of particle motion in the wave.
Stretching decreases velocity. The size of velocity changes is proportional to stress.

The dislocation part of velocity changes characterizes the dislocation lattice of
the crystal and its value depends on density and average free length of the dislocation
loops. This means that the value of dislocation velocity changes depends on the
degree of plastic deformation, recovery after deformation, content of point defects, as
well as frequency and amplitude of ultrasonic waves. The direction of dislocation
velocity changes remains the same when the sign of stress is changed.

The value is considered constant for a given sort of material in ultrasonic
measurements of stress. Results of this research indicate that such an approach
should be limited. In practice a steel sample kept at room temperature during several
days from cold plastic deformation exhibits full recovery of the modulus and decay of
the dislocation component of velocity changes due to stress. In order to state
whether a given material contains the dislocation component of velocity change, the
symmetry criterion during stress direction change and frequency test can be used.

References

[2] R. E. Green, Treatise on materials science and technology, V. 3 — Ultrasonic investigation of
[3] В. М. Бобренко, А. М. Кущенко, Акустическая тензометрия I, физические основы,
Дефектоскопия, 2, (1980); Акустическая тензометрия II, Методы и устройства,
Дефектоскопия, 12 (1980).
[8] A. Granato, K. Lücke, Theory of mechanical damping due to dislocations, J. Appl. Phys., 27, 583 and
789 (1956).
[13] A. Brokowski, J. Deputat, Ultrasonic testing of the influence of degree of deformation in iron, (in