

DESIGN OF ULTRASONIC PROBES FOR MEDICAL DIAGNOSTICS

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The Mason's equivalent circuit of the piezoelectric transducer is modified in order to investigate separately the influence of parameters of piezoelectric material as well as electrical and acoustical elements applied in the ultrasonic probe. All values transferred to the mechanical side of the electromechanical transformer let us introduce the relative values both electrical and acoustical parameters. The mechanical and dielectric losses of the transducer are taken into consideration.

The influence of various parameters of the probe on input admittance and reflected from an ideal reflector pulses is shown.

Schemat zastępczy Masona przetwornika piezoelektrycznego został zmodyfikowany w sposób umożliwiający badanie oddzielnie wpływu zarówno parametrów piezoelektrycznego materiału, jak również elektrycznych i akustycznych elementów zastosowanych w głowicy. Wszystkie wielkości przeniesiono na stronę mechaniczną transformatora elektromechanicznego, co pozwoliło na wprowadzenie wartości względnych zarówno elektrycznych, jak i akustycznych parametrów. W układzie zastępczym uwzględniono straty mechaniczne i dielektryczne przetwornika.

Pokazano wpływ różnych parametrów głowicy na admitancję wejściową i impulsy odbite od idealnego reflektora.

Notation

- A surface of transducer
- c_a velocity of ultrasonic wave in medium loading the back surface of transducer
- c_b velocity of ultrasonic wave in investigated medium
- c_p velocity of ultrasonic wave in transducer
- c_{01} velocity of ultrasonic wave in first matching layer
- c_{02} velocity of ultrasonic wave in second matching layer
- C_0 clamped capacitance of transducer
- C_m capacitance in equivalent series resonance system of transducer
- d thickness of transducer
- d_{01} thickness of first matching layer

- d_{02} thickness of second matching layer
 E_t voltage of transmitter
 E'_t relative voltage of transmitter
 E_r voltage of pulse reflected from an ideal reflector
 E'_r relative voltage of pulse reflected from an ideal reflector
 f frequency
 f_e electric resonance frequency of transducer
 f_m mechanical resonance frequency of transducer
 F_t acoustic force radiated by the transducer
 F_r acoustic force reflected from ideal reflector
 k parameter describing capacitance in compensating circuit
 k_t electromechanical coupling coefficient
 l parameter describing series inductance in compensating circuit
 l_c length of cable
 L_m inductance in equivalent series resonance circuit of transducer
 m parameter describing parallel inductance in compensating circuit
 N turns ratio of electromechanical transformer
 n parameter describing thickness of first matching layer
 p parameter describing thickness of second matching layer
 Q_m mechanical factor of transducer
 R_a relative acoustic resistance of back loading of transducer
 R_b relative acoustic resistance of investigated medium
 R_e relative acoustic resistance of dielectric losses
 R_m relative acoustic resistance of mechanical losses
 R_{oc} characteristic impedance of cable
 R_p acoustic resistance of transducer
 R_r resistance of receiver
 R'_r relative resistance of receiver
 R_t resistance of transmitter
 R'_t relative resistance of transmitter
 R_A acoustic resistance of back loading of transducer
 R_B acoustic resistance of investigated medium
 R_F acoustic impedance of transducer at resonance frequency f_e
 R_{01} relative acoustic impedance of first matching layer
 R_{02} relative impedance of second matching layer
 t time
 T_m period of transducer mechanical vibrations
 w_c coefficient describing characteristic impedance of cable
 w_m coefficient describing resistance of mechanical losses in terms of frequency
 x relative frequency
 x_e relative frequency of electric resonance
 x_m relative frequency of mechanical resonance
 X_{in} imaginary part of relative mechanical impedance of transducer
 Y_{in} relative input admittance of transducer
 Z_E input impedance of transducer
 Z_{in} relative input impedance of transducer
 Z_M mechanical impedance of transducer
 Z_m relative mechanical impedance of transducer
 Z_{Ck} relative capacitance impedance in compensating circuit
 Z_{L1} relative impedance of a series coil in compensating circuit
 Z_{Lm} relative impedance of series coil in compensating circuit
 β phase constant of lines (matching layers)

δ_e	angle of dielectric losses
δ_m	angle of mechanical losses
ϵ^S	dielectric constant
φ_c	coefficient describing length of cable
λ_e	length of acoustic wave for electric resonance of transducer
λ_m	length of acoustic wave for mechanical resonance of transducer
λ_{el}	length of electromagnetic wave in cable
ρ_a	density of back loading of transducer
ρ_b	density of investigated medium
ρ_p	density of transducer
ρ_{01}	density of first matching layer
ρ_{02}	density of second matching layer
ω	pulsation
ω_e	pulsation of electric resonance of transducer
ω_m	pulsation of mechanical resonance of transducer

1. Introduction

Ultrasonic medical diagnostics have been developing during the last 30 years. A clear progress can be noted in the past several years. It results from computerization on one hand and perfection of ultrasonic transmitting-receiving systems on the other. For example an increase of general sensitivity led to an increase of operating frequency of ultrasonographs for abdominal examinations from 2.5 MHz to 3.5 MHz, and this, in turn, improved resolution. The progress in this field is conditioned by the construction of adequate ultrasonic probes. Sensitivity is increased by eliminating or decreasing loading of the back surface, by using layers matching acoustic impedance of the acoustic transducer to the human body and by matching electric impedance of the probe to that of echograph transmitting-receiving systems.

The following quantities influence the shape and size of received ultrasonic pulses: voltage exciting the transducer, electric and acoustic systems, parameters of piezoelectric materials and the transducer dimensions.

In order to investigate the influence of these parameters separately, the transducer equivalent circuit was modified by introducing relative quantities and by including dielectric and mechanical losses (what is essential when piezoelectric film is applied). Also quantities describing the compensating system, cable, transmitter and receiver were introduced into the model.

Designers of ultrasonic probes generally tend to achieve flat amplitude and linear phase characteristics of the transmitting-receiving path and to minimize losses due to processing. However it is difficult to describe the pulse response dependence on amplitude-phase characteristics, therefore the influence of individual parameters of the system on its operation was usually illustrated with pulses reflected from an ideal reflector.

The analysis is carried out on the basis of the four-terminal theory and FFT techniques.

2. Equivalent circuit of an ultrasonic probe

The equivalent circuit of an ultrasonic probe is presented in Fig. 1. Many authors [8], [15], [16] apply the KLM equivalent circuit. However, here we accepted Mason's equivalent circuit [13], because in the KLM model the turns ratio of an

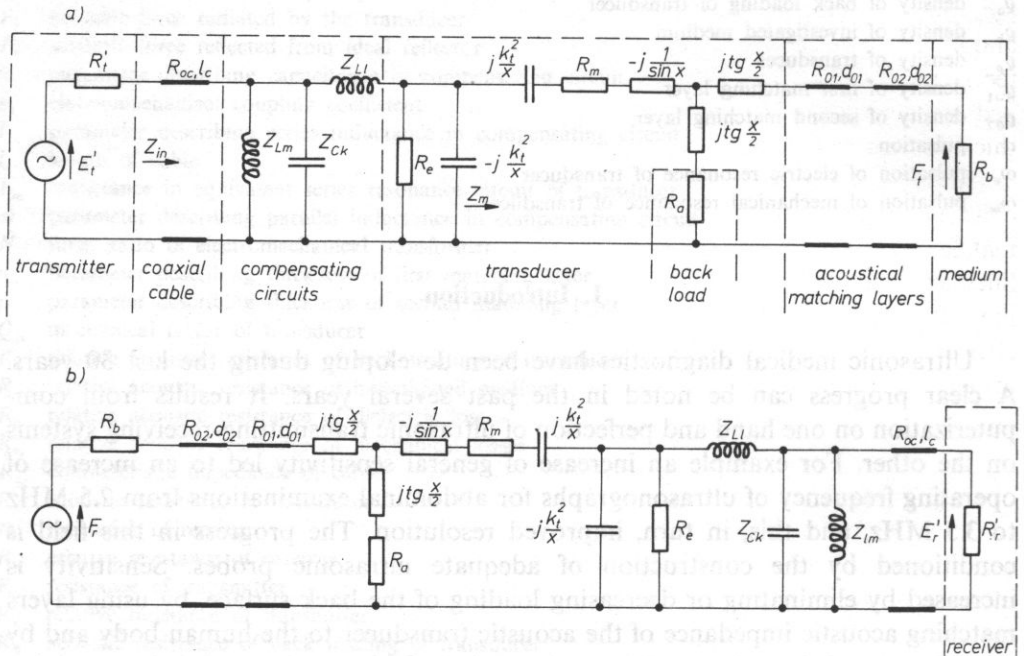


FIG. 1. Equivalent circuit of ultrasonic probe: a – transmitting, b – receiving (parameters described in paper)

electromechanical transformer is frequency dependent. It is possible to calculate the transducer input impedance and the electric resonance frequency f_e – electromechanical coupling coefficient k_t relationship [10] directly from Mason's model.

All quantities describing the probe and input transmitting-receiving systems have been transferred in the model to the mechanical side of the electromechanical transformer (comparative with Fig. 9), and at the same time relative values of parameters were introduced:

- frequency $x = \Pi f / f_m$
- acoustic resistance of transducer $R_p / R_p = 1$ where $R_p = A Q_p c_p$
- resistance of dielectric and mechanical losses R_e and R_m
- acoustic resistance of back loading $R_a = A Q_a c_a / R_p$
- acoustic resistance of investigated medium $R_B = A Q_b c_B / R_p$
- parameters describing matching layers R_{01} , n and R_{02} , p

- parameters describing the coaxial cable R_{0c}, l
- impedances of compensating system Z_{Ll}, Z_{Ck}, Z_{Lm}
- voltage of transmitter $E'_t = E_t N$
- voltage of pulse reflected from an ideal reflector $E'_r = E_r N$
- resistance of transmitter and receiver $R'_t = R_t N^2 / R_p, R'_r = R_r N^2 / R_p$, where f_m – frequency of transducer mechanical resonance, A – surface of transducer, $q_p c_p, q_a c_a, q_b c_b$ – acoustic resistances of transducer, loading of transducer back surface and investigated medium, respectively, R_t, R_r – input impedances of transmitter and receiver, $N^2 = 2 k_t^2 f_m C_0 R_p$ turns ratio of electromechanical transformer, C_0 – clamped capacitance of transducer, k_t – electromechanical coupling coefficient for thickness vibrations, E_N – voltage of inciding wave, E_r – voltage of reflected wave.

Additional elements can be included in the probe equivalent circuit (subject to its construction), such as e.g. layers matching back loading to transducer, or transformer matching the electrical impedance of the probe to the transmitting-receiving circuit. Such an equivalent circuit with all these relative quantities can be used to investigate the influence of individual parameters of piezoelectric ceramics, as well as of other elements of the probe, on the size and shape of reflected pulse, independently of the transducer diameter and its resonance frequency.

2.1 Dielectric and mechanical losses of the transducer

L. F. BROWN and D. L. CARLSON [1] have investigated polymer piezoelectric films with high dielectric and mechanical losses, and proved that the actual input impedance of the transducer can be modelled with 1% accuracy when resistances of these losses in the form as in Fig. 1 are included in Mason's model. Thus, according to results of their papers, relative quantities can be introduced as follows.

Dielectric losses are described with $\text{tg } \delta_e$ which represents the ratio of the conduction current to the displacement current in a dielectric, so

$$\text{tg } \delta_e = \frac{1}{R_e \omega C_0}. \quad (1)$$

Because in the modified Mason's model (Fig. 1) the impedance of C_0 is represented as

$$\frac{1}{\omega C_0} \Rightarrow \frac{k_t^2}{x}$$

then the resistance expressing losses is

$$R_e = \frac{k_t^2}{x \text{tg } \delta_e}. \quad (2)$$

In order to introduce element R_m , which describes mechanical losses, let us consider mechanical impedance Z_m of a not loaded transducer ($R_a = R_b = R_{01} = R_{02} = 0$).

It will have the form as in Fig. 2a, b.

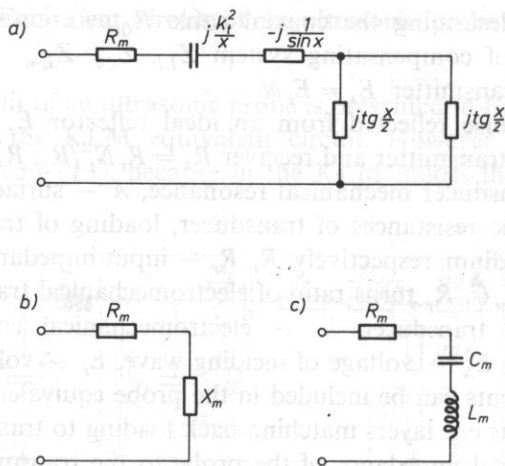


Fig. 2. Equivalent circuit of mechanical part of transducer (not loaded): a and b according to Mason's model, c near electric resonance

Hence, we can write

$$Z_m = R_m + jX_m, \quad (3)$$

where

$$X_m = \frac{k_t^2}{x} - \frac{1}{\sin x} + \frac{1}{2} \operatorname{tg} \frac{x}{2} = \frac{k_t^2}{x} - \frac{1}{2} \operatorname{ctg} \frac{x}{2}. \quad (4)$$

The electric resonance of a transducer occurs at frequency, for which $X_m = 0$. This leads to expression

$$k_t^2 = \frac{x_e}{2} \operatorname{ctg} \frac{x_e}{2}, \quad (5)$$

where $x_e = f_e/f_m$, f_e — frequency of electric resonance. As we can see, the relative frequency of electric resonance of a not loaded transducer is a function of the electromechanical coupling coefficient only (Fig. 3).

Near electric resonance mechanical impedance of the transducer can be substituted with good accuracy with a series resonance circuit R_m , C_m , L_m (Fig. 2c). Thus

$$X_m = \omega L_m - \frac{1}{\omega C_m}. \quad (6)$$

The following expressions can be written for this circuit

$$Q_m = \frac{1}{R_m \omega_e C_m}, \quad (7)$$

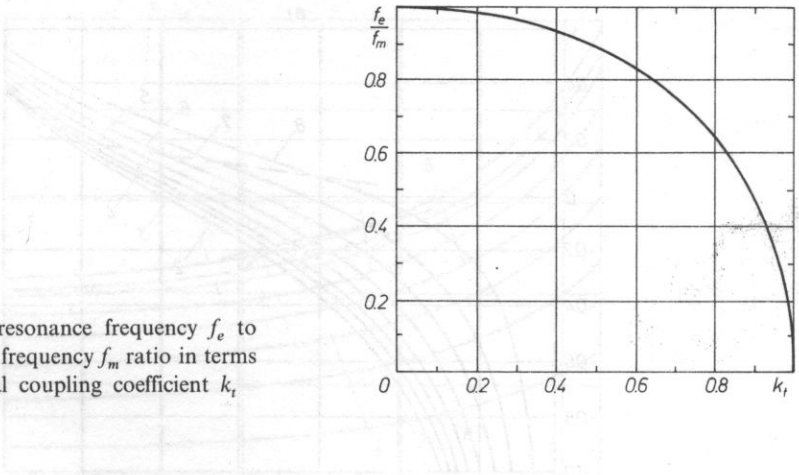


FIG. 3. The electric resonance frequency f_e to mechanical resonance frequency f_m ratio in terms of electromechanical coupling coefficient k_t

where $\omega_e = 2\pi f_e$

$$\omega_e = 1/\sqrt{L_m C_m} \tag{8}$$

and thus

$$X_m = \frac{1}{C_m \omega_e} \left(\frac{\omega}{\omega_e} - \frac{\omega_e}{\omega} \right). \tag{9}$$

Figure 4 presents values of X_m calculated in accordance with formula (4) (Fig. 4a) and formula (9) (Fig. 4b). Values X_m have been equalized here for frequencies x_e and $1.25 x_e$. As we can see these curves are identical with an accuracy of the drawing in the frequency range of the relative band $\Delta f/f_e \approx 0.5$.

If we accept, as L. F. BROWN and D. L. CARLSON [1] did, that factor Q_m can be assumed independent of frequency, then it will be the inverse of the mechanical losses tangent

$$Q_m = 1/\text{tg} \delta_m. \tag{10}$$

From expressions (4), (5), (7) and (10) we obtain the formula for R_m in the following form

$$R_m = W_m \text{tg} \delta_m, \tag{11}$$

where

$$W_m = X_m(x)/(x/x_e - x_e/x) = \frac{1}{2} \left(\frac{x_e}{x} \text{ctg} \frac{x_e}{2} - \text{ctg} \frac{x}{2} \right) / (x/x_e - x_e/x). \tag{12}$$

Figure 5 presents diagrams of coefficient W_m in terms of frequency for various k_t values. Points mark frequencies of electric resonance (f_e/f_m). Knowing the value of

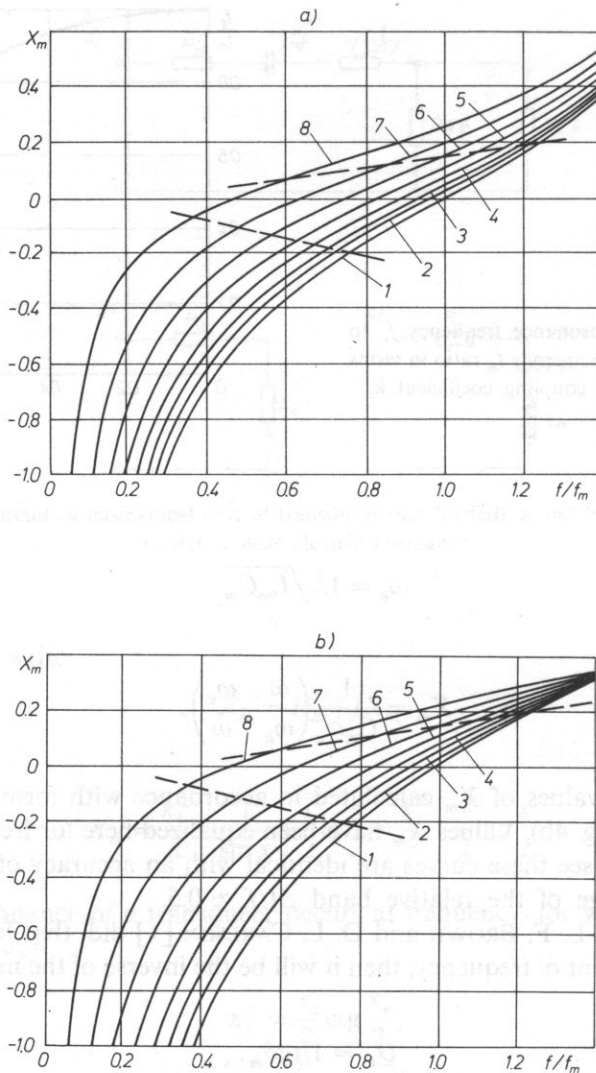


FIG. 4. The imaginary part of mechanical impedance of transducer not loaded in terms of frequency to mechanical resonance frequency f_m ratio, resulting from: a — Mason's equivalent circuit, b — equivalent series resonance circuit for: 1 — $k_t = 0.1$, 2 — $k_t = 0.3$, 3 — $k_t = 0.4$, 4 — $k_t = 0.5$, 5 — $k_t = 0.6$, 6 — $k_t = 0.7$, 7 — $k_t = 0.8$, 8 — $k_t = 0.9$. Broken lines mark the range with frequency band $\Delta f/f_e$ equal to 0.5

the mechanical losses tangent $\text{tg} \delta_m$, we can include the resistance of losses R_m in the equivalent transducer circuit Fig. 1.

It should be noted that a different dependence of R_e and R_m on frequency could be used in formulae (2) and (11) — a dependence defining losses in various piezoelectric materials more accurately.

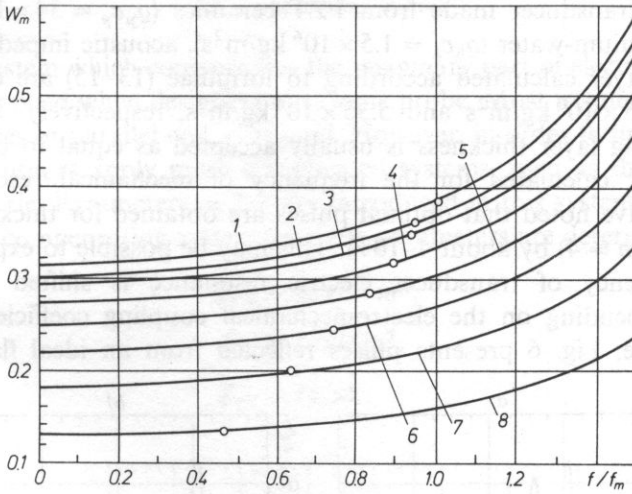


FIG. 5. W_m coefficient (describing mechanical losses according to expression (12)) in terms of frequency to mechanical resonance frequency f_m ratio for: 1 - $k_t = 0.1$, 2 - $k_t = 0.3$, 3 - $k_t = 0.4$, 4 - $k_t = 0.5$, 5 - $k_t = 0.6$, 6 - $k_t = 0.7$, 7 - $k_t = 0.8$, 8 - $k_t = 0.9$. Points mark the electric resonance frequencies f_e/f_m

2.2 Acoustic matching layers

Acoustic layers matching the acoustic impedance of the transducer to the impedance of investigated medium (or back loading of transducer) are described with two parameters - acoustic characteristic impedance of line $R_{01} = Aq_{01}c_{01}/R_p$ and parameter n corresponding with layer thickness constant phase line β in accordance with formula $n = \lambda_m/d_{01}$, where λ_m - wave length for transducer mechanical resonance frequency, d_{01} - thickness of layer (length of line). Because $\beta = 2\pi d_{01}/\lambda$ and $\lambda = c_{01}/f$, $\lambda_m = c_{01}/f_m$, then $\beta = 2\pi n$.

Parameters of the second matching layer, R_{02} and p are determined by analogy. Naturally, an arbitrary number of layers can be introduced into the system.

Characteristic impedances of lines are generally determined from Czebyszew's formulae (geometric mean) [3]. Other authors calculate acoustic impedance of matching layers on the basis of various premises. DE SILETS et al. [15], [16] have proved that the shape of received pulses is optimal when the acoustic resistance of matching layers is calculated according to the binominal criterion. SOUQUET et al. [17], [4] have derived formulae for acoustic impedance of these layers assuming an equality of Q factors of the electric and mechanical branch of the equivalent circuit.

Formulae for acoustic impedance of a single matching layer are as follows

$$\text{according to CZEBYSZEW} \quad q_{01}c_{01} = \sqrt{(q_p c_p)(q_b c_b)}, \quad (13)$$

$$\text{according to DE SILETS} \quad q_{01}c_{01} = \sqrt[3]{(q_p c_p)(q_b c_b)^2}, \quad (14)$$

$$\text{according to SOUQUET} \quad q_{01}c_{01} = \sqrt[3]{2(q_p c_p)(q_b c_b)^2}, \quad (15)$$

And so, for a transducer made from PZT ceramics ($\rho_p c_p = 34 \times 10^6 \text{ kg/m}^2\text{s}$) and investigated medium-water ($\rho_b c_b = 1.5 \times 10^6 \text{ kg/m}^2\text{s}$), acoustic impedances $\rho_{01} c_{01}$ of the matching layer calculated according to formulae (13–15) are equal to $7.14 \times 10^6 \text{ kg/m}^2\text{s}$, $4.25 \times 10^6 \text{ kg/m}^2\text{s}$ and $5.35 \times 10^6 \text{ kg/m}^2\text{s}$, respectively.

The matching layer thickness is usually accepted as equal to one fourth of the wave length — calculated for the frequency of mechanical resonance. Certain authors [16] have noted that optimal pulses are obtained for thicknesses of layers exceeding $\lambda_m/4$ ($n = 4$) by about 4–10%. This may be possible to explain by the fact that the frequency of transducer electric resonance is shifted towards lower frequencies, depending on the electromechanical coupling coefficient k_t .

For example, Fig. 6 presents pulses reflected from an ideal flat reflector, for

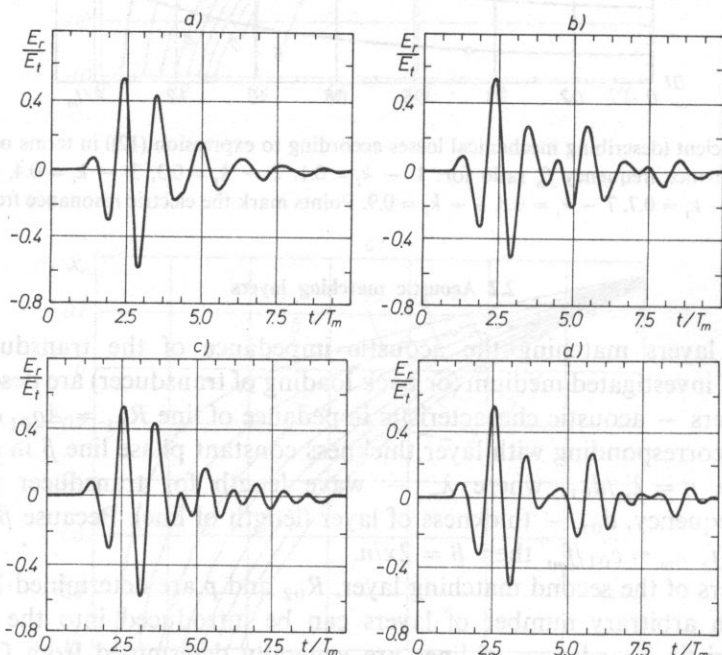


FIG. 6. Pulses reflected from ideal reflector for PZT I ceramic transducer with two matching layer for excitation voltage $E_i = \delta(t)$, $k_t = 0.5$, $R_a = 0$, $R_b = 0.044$

- a) $R_{01} = 0.262$, $R_{02} = 0.069$, $n = p = 4$
- b) $R_{01} = 0.36$, $R_{02} = 0.122$, $n = p = 4$
- c) $R_{01} = 0.262$, $R_{02} = 0.069$, $n = p = 3.55$
- d) $R_{01} = 0.36$, $R_{02} = 0.122$, $n = p = 3.55$

$$\text{where } T_m = 1/f_m$$

a transducer from PZT ceramics with two matching layers with acoustic impedances calculated according to the binomial criterion (a, c) and Czebyszew's formulae (b, d), and thickness equal to one fourth of wave length calculated for mechanical resonance frequency $\lambda_m/4$ (a, b) and electric resonance frequency λ_e (c, d). We can see that these parameters only slightly influence the pulse amplitude, while it has significant influence on its shape.

2.3. Compensating system

Usually a system which compensates the imaginary part of electric impedance or admittance is applied when designing ultrasonic probe. Most frequently the simplest system — a series or parallel coil — is used. However, in order to improve the pulse shape certain authors apply more complicated systems, such as the one shown in Fig. 1 for example. Parameters m , l , k are introduced in this system. Impedances of elements of the compensating system for these parameters are determined as follows

$$Z_{Lm} = k_t^2 x / \pi^2 m^2,$$

$$Z_{Ll} = k_t^2 x l^2 / \pi^2,$$

$$Z_{Ck} = k_t^2 / x k.$$

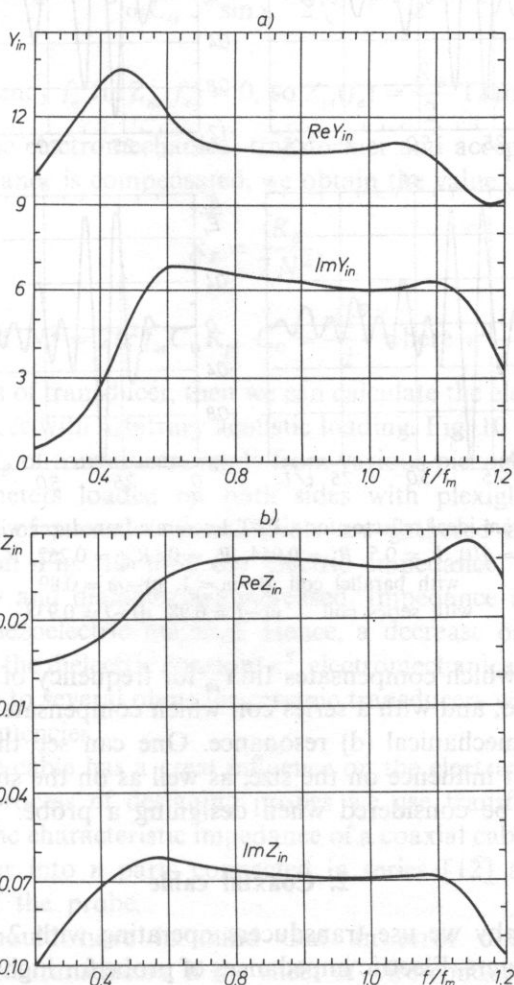


FIG. 7. Diagrams: a) — of relative input admittance Y_{in} , b) — of relative input impedance Z_{in} of transducer with two matching layers. $k_t = 0.5$, $R_b = 0.044$, $R_a = 0$, $R_{01} = 0.262$, $R_{02} = 0.069$. $n = p = 3.55$

The inductance of a parallel coil Z_m compensates the transducer clamped capacitance for frequency equal to mf_m , i.e. for transducer mechanical resonance $m = 1$, while for electric resonance $m = f_e/f_m$.

Figure 7 presents the admittance (a) and impedance (b) of a transducer from PZT I ceramics with matching layers, calculated according to the binomial criterion, with thickness equal to one fourth of wave length for electric resonance frequency. These diagrams were used to calculate parameters m and l .

In Fig. 8 we can see pulses reflected from an ideal reflector for a probe as in Fig. 7,

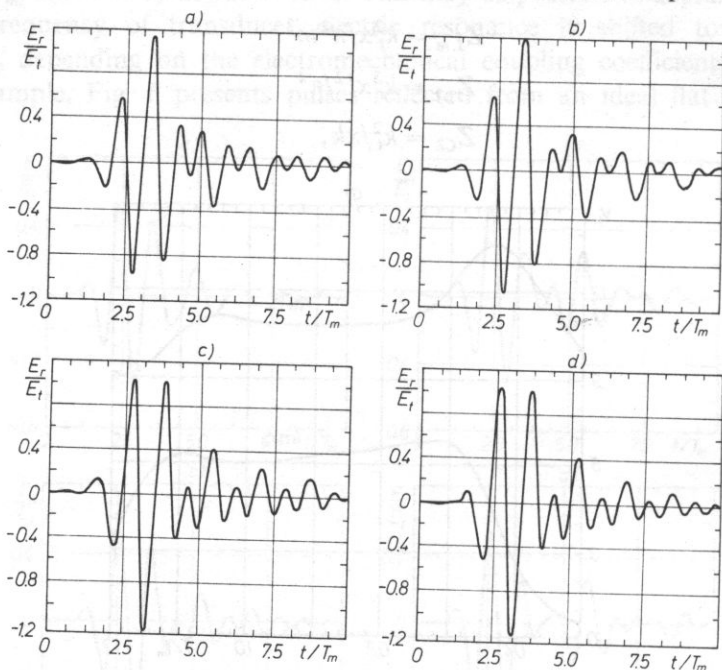


Fig. 8. Pulses reflected from ideal reflector for a PZT I ceramic transducer with two matching layers for excitation voltage $E_i = \delta(t)$, $k_t = 0.5$, $R_b = 0.044$, $R_a = 0$, $R_{01} = 0.262$, $R_{02} = 0.069$, $n = p = 3.55$ with parallel coil a) $-m = 1$, b) $-m = 0.89$ with series coil c) $-l = 0.98$, d) $-l = 0.938$

with a parallel coil which compensates $\text{Im}Y_{in}$ for frequency of a mechanical (a) and electric (b) resonance; and with a series coil which compensates $\text{Im}Z_{in}$ for frequency of electric (c) and mechanical (d) resonance. One can see that the compensating system has a distinct influence on the size, as well as on the shape, of the pulse and therefore it has to be considered when designing a probe.

2. Coaxial cable

In ultrasonography we use transducers operating with 2–10 MHz frequencies and 3–50 mm diameters. Electric impedances of probe for high frequencies and large diameters are equal to several ohms. Let us consider for example a transducer loaded symmetrically with acoustic impedance $R_B = A Q_b c_b$ (Fig. 9).

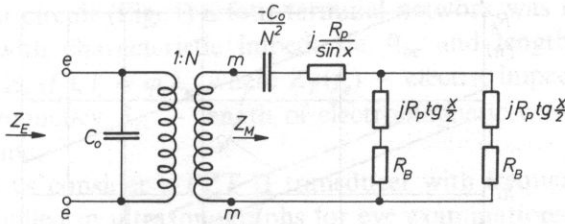


FIG. 9. Equivalent circuit of transducer loaded symmetrically with acoustic impedance R_B

The transducer mechanical impedance is then expressed by

$$Z_M = -j \frac{N^2}{\omega C_0} + j \frac{R_p}{\sin x} + \frac{1}{2} \left(j R_p \operatorname{tg} \frac{x}{2} + R_B \right).$$

For resonance frequency f_e $\operatorname{Im} Z_m(f_e) = 0$, so $Z_m(f_e) = \frac{R_B}{2}$. Transposing all terms to the electric side of the electromechanical transformer and accepting the assumption that clamped capacitance is compensated, we obtain the value of electric impedance

$$R_E = \frac{R_B}{2N^2}.$$

If we remember that $N^2 = 2k_t^2 f_m C_0 R_p$, $C_0 = \frac{A \varepsilon^S}{d}$ where ε^S – dielectric constant, $d = \lambda_m/2$ – thickness of transducer, then we can calculate the electric impedances for an arbitrary transducer with arbitrary acoustic loading. Fig. 10 presents diagrams of electric impedance R_E of transducers made from various piezoelectric materials with 5 and 20 mm diameters loaded on both sides with plexiglass ($\rho_b c_b = 3.2 \times 10^6$ kg/m²s). Parameters of piezoelectric materials are gathered in Table 1.

We can see from Fig. 10 that the electric impedance decreases when the transducer frequency and diameter are increased. Impedance is also influenced by parameters of the piezoelectric material. Hence, a decrease of impedance accompanies an increase of the dielectric constant ε^S , electromechanical coupling coefficient k_t . And so, it is equal to several ohms for ceramic transducers with 20 mm diameters and 5–10 MHz frequencies.

In these cases the cable has a great influence on the electric characteristic of the transducer. In the process of designing probes we use transformers matching the probe resistance to the characteristic impedance of a coaxial cable; or, sometimes, we divide the transducer into n parts connected in series [12] and obtain n^2 times greater resistance of the probe.

However, we should have in mind that an error may be made during measurements of such transducers, if the effect of wire conductors is not included. The influence of these conductors has to be also considered during the process of designing ultrasonic probes.

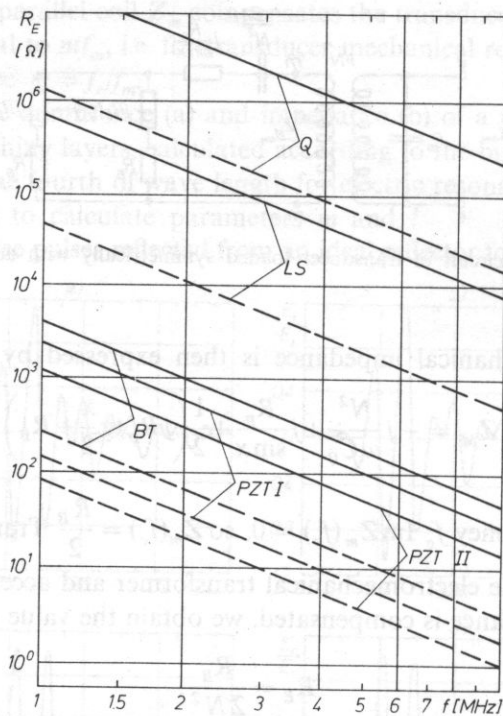


Fig. 10. Electric impedance of transducers loaded on both sides with plexiglass ($\rho_b c_b = 3.2 \cdot 10^6 \text{ kg/m}^2\text{s}$) in terms of their resonance frequencies (compensated clamped capacitance) Q — quartz, LS — lithium sulphate, BT — titanate ceramics, PZT I, PZT II — PZT type ceramics, full line — diameter of transducer — 5 mm, broken line — diameter of transducer — 20 mm

Table 1. Parameters of piezoelectric materials

Material		Density $\times 10^3 \text{ kg/m}^3$	Relative dielectric constant ϵ^S	Coupling coefficient k_t
Quartz	*	2.65	4.6	0.095
Lithium sulphate	*	2.06	10.3	0.38
Barium titanate	*	5.6	1200	0.3
PZT I	**	7.5	500	0.5
PZT II	**	7.5	500	0.7

* According to G. BRADFIELD, Ultrasonics 1970 [2].

** According to measurements of domestic ceramics prod. CERAD.

In the equivalent circuit (Fig. 1) a four-terminal network was included to define the coaxial cable with characteristic impedance R_{oc} and length l_c , expressed as follows: $R_{oc} = w_c R_e Z_E(f_e)$, $l_c = \varphi_c \lambda_{el}$ where $Z_E(f_e)$ — electric impedance of probe for electric resonance frequency, λ_{el} — length of electromagnetic wave in the cable, w_c and φ_c — coefficients.

For example let us consider a PZT II transducer with frequency $f_e = 10$ MHz, diameter 20 mm (applied in ultrasonographs for eye examinations), loaded on both sides with plexiglass. Electric impedance equal to $R_E = 1\Omega$ results from the diagram in Fig. 10. If we consider wire conductors as a long line with wave impedance of $R_{oc} = 300\Omega$ and $\lambda_{el} = 30$ m, then $w_c = 300$, $\varphi_c = 0.0005$ (for connectors with length $l_c = 1.5$ cm) and $\varphi_c = 0.0001$ (for $l_c = 3$ cm). It becomes clear that the cable influences the admittance value, as well as the shift of transducer resonance frequency (Fig. 11).

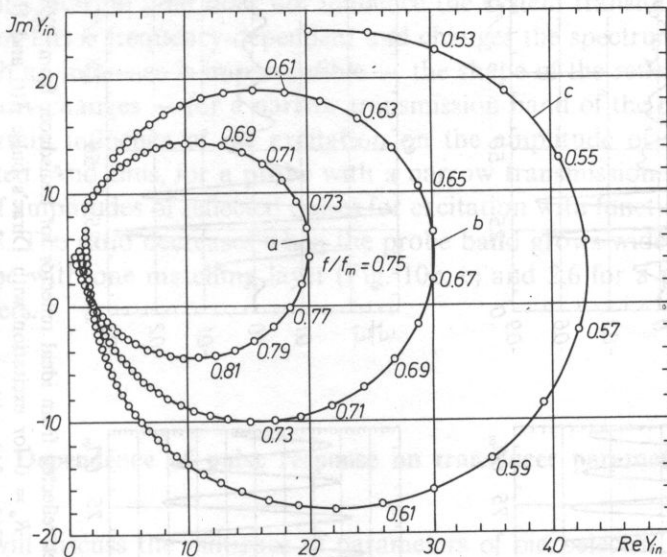


FIG. 11. Circles of relative input admittance of transducer from PZT II ceramics, with diameter equal to 20 mm and frequency equal to 10 MHz, loaded on both sides with plexiglass, $k_t = 0.7$, $R_a = R_b = 0.1$

- a) — without cable supply conductors,
 - b) — with cable $l_c = 1.5$ cm, $\varphi_c = 0.0005$, $w_c = 300$,
 - c) — with cable $l_c = 3$ cm, $\varphi_c = 0.0001$, $w_c = 300$,
- f/f_m — parameter

3. Transmitting-receiving system

The design of the transmitting-receiving system and the transmitter, in particular, significantly influences the size and shape of the pulse transmitted, as well as the received pulse. However, it is much easier to use a simple system, such as in Fig. 1, in the process of designing ultrasonic probes. Here, the transmitter is presented as

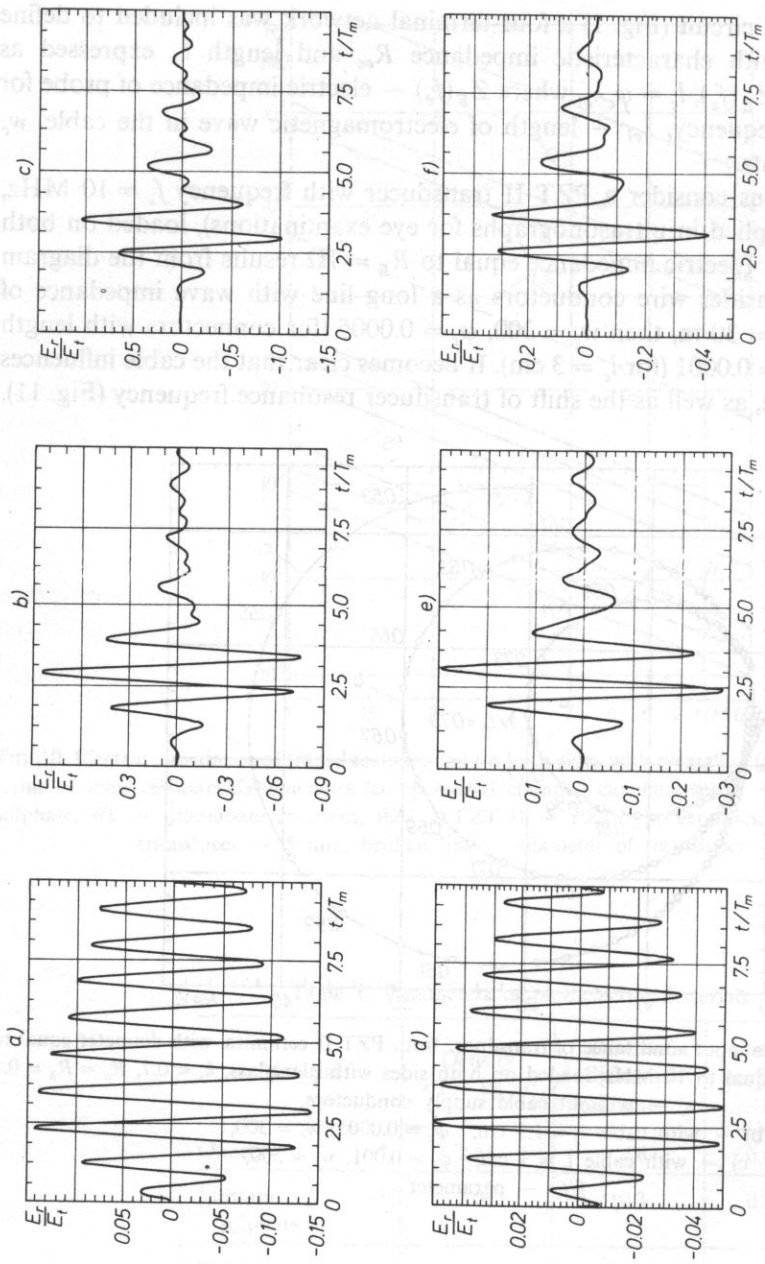


Fig. 12. Pulses reflected from ideal reflector for transducer from PZT I ceramics, $k_t = 0.5$, $R_b = 0.044$, $R_o = 0$ for excitation with Dirac's pulse $\delta(t)$ and Heviside pulse $1(t)$

- a) - $E(t) = \delta(t)$, without matching layers
- b) - $E(t) = \delta(t)$, with one matching layer $R_{o1} = 0.122$, $n = 3.55$
- c) - $E(t) = \delta(t)$, with two matching layers $R_{o1} = 0.262$, $R_{o2} = 0.069$, $n = p = 3.55$
- d) - $E(t) = 1(t)$ without layers
- e) - $E(t) = 1(t)$ with one layer $R_{o1} = 0.122$, $n = 3.55$
- f) - $E(t) = 1(t)$ with two layers $R_{o1} = 0.262$, $R_{o2} = 0.069$, $n = p = 3.55$

source E'_t and resistance R'_t ; receiver as resistance R'_r , as it was described in Sect. 2. It is easier to design the ultrasonic probe, including the ratio of probe impedance to source and receiver impedance when electric quantities are transferred to the mechanical side of the electromechanical transformer.

The excitation voltage can be defined with any function containing Fourier's transform. Most frequently Heviside's function $1(t)$ or Dirac's function $\delta(t)$ with unitary amplitude is used in the course of designing probes. Figure 12 presents pulses reflected from an ideal reflector for a PZT I transducer with a parallel coil, without matching layers (a, d) with one matching layer (b, e) and with two matching layers (c, f), excited with voltage $\delta(t)$ (a, b, c) and $1(t)$ (d, e, f).

The application of matching layers widens the probe transmission band. This is why here we see a change of the reflected pulse shape (see Figs. 10b and 10e, 10c and 10f) depending on the used excitation voltage. This is justifiable because function $\delta(t)$ has a constant spectrum (and does not influence the system transmission function), while spectrum $1(t)$ is frequency-dependent and changes the spectrum of the system response. Such an influence is imperceptible — the shape of the reflected pulse does not undergo any changes — for a narrow transmission band of the probe (Fig. 10a, d). Also a certain influence of the excitation on the amplitude of reflected pulses should be noted. And thus, for a probe with a narrow transmission band (Fig. 10a, d), the ratio of amplitudes of reflected pulses for excitation with function $\delta(t)$ and $1(t)$ is equal to 2.8. The ratio decreases when the probe band grows wider. It is equal to 2.7 for a probe with one matching layer (Fig. 10b, e) and 2.6 for a probe with two matching layers.

4. Dependence of pulse response on transducer parameters

Here we will discuss the influence of parameters of piezoelectric materials, $q_p c_p$ and k_t , on the shape and size of pulses reflected from an ideal reflector, for probes with and without matching layers, and with and without a parallel compensating coil. Transducers from PZT type ceramics lead metaniobate ceramics and quartz (only for comparison, because quartz is not used in ultrasonography due to difficulties with electric matching) were analysed.

The following assumptions were made to eliminate from discussion the influence of parameters of the transmitting-receiving system and matching layers:

- the excitation voltage is defined by Dirac's function $\delta(t)$
- $R_t = 0.01 \ 1/Re \ Y_{in}(x_e) \quad R_r = 10 \ 1/Re \ Y_{in}(x_e)$
- acoustic impedances of matching layers were calculated according to the binomial criterion
- the thickness of matching layers is equal to $1/4$ of wave length for electric resonance

- the inductance of the parallel coil compensates the imaginary part of the input admittance for electric resonance
- acoustic impedance of examined medium (soft tissue) $\rho_b c_b = 1.5 \times 10^6 \text{ kg/m}^2\text{s}$
- the back surface of the transducer is not loaded ($R_a = 0$).

4.1. Reflected pulses, conclusions

The size of pulses for quartz is by an order of magnitude smaller than of those for ceramic materials in a case of transducers without matching layers and without a coil (Fig. 13a, b, c, d). The amplitude is first of all influenced by the electromechanical coupling coefficient k_t . With a parallel coil added (Fig. 13e, f, g, h), sizes of pulses for quartz and lead metaniobate ceramics are comparable, while for PZT ceramics they are twice as small and longer. This can be explained by better acoustic matching of quartz and lead metaniobate ceramics to the examined medium than in case of PZT.

For transducers with one matching layer and without a coil (Fig. 14a, b, c, d) the shortest pulse is observed for quartz. This is an evidence for very good acoustic matching. It is worth noting that the pulse amplitude for PZT II is four times larger than for lead metaniobate ceramics. This means that k_t has greater influence on the pulse size when transducers are acoustically matched. The attachment of a coil (Fig. 14e, f, g, h) has the greater effect, the smaller coefficient k_t is. For quartz the amplitude increases about 10 times yet the shape of the pulse worsens; for PZT II it practically does not change at all.

The application of two matching layers (Fig. 15) makes the amplitude smaller and worsens the pulse shape for quartz, hardly changes the pulse for lead metaniobate ceramics; increases the amplitude for PZT which has the greatest acoustic resistance. This is strictly related to acoustic matching of the transducer itself to the medium.

To sum up we can state that the pulse size and shape is influenced by the transducer acoustic resistance, as well as by the electromechanical coupling coefficient k_t . The choice of ceramics for the design of ultrasonic probes should depend on the possibility of electric matching (in ultrasonography frequency and, frequently, diameter of the transducer — and the electric impedance as a result — is imposed depending on the examined organ) and construction restrictions (greater difficulties with making two matching layers). However, it should be noted that largest amplitudes are achieved for PZT ceramics with two matching layers.

5. Conclusions

The proposed equivalent circuit of an ultrasonic probe with all quantities describing electric, as well as mechanical parameters of the transducer transferred to one side of the electromechanical transformer can be used to design probes, taking into account the influence of all of these quantities.

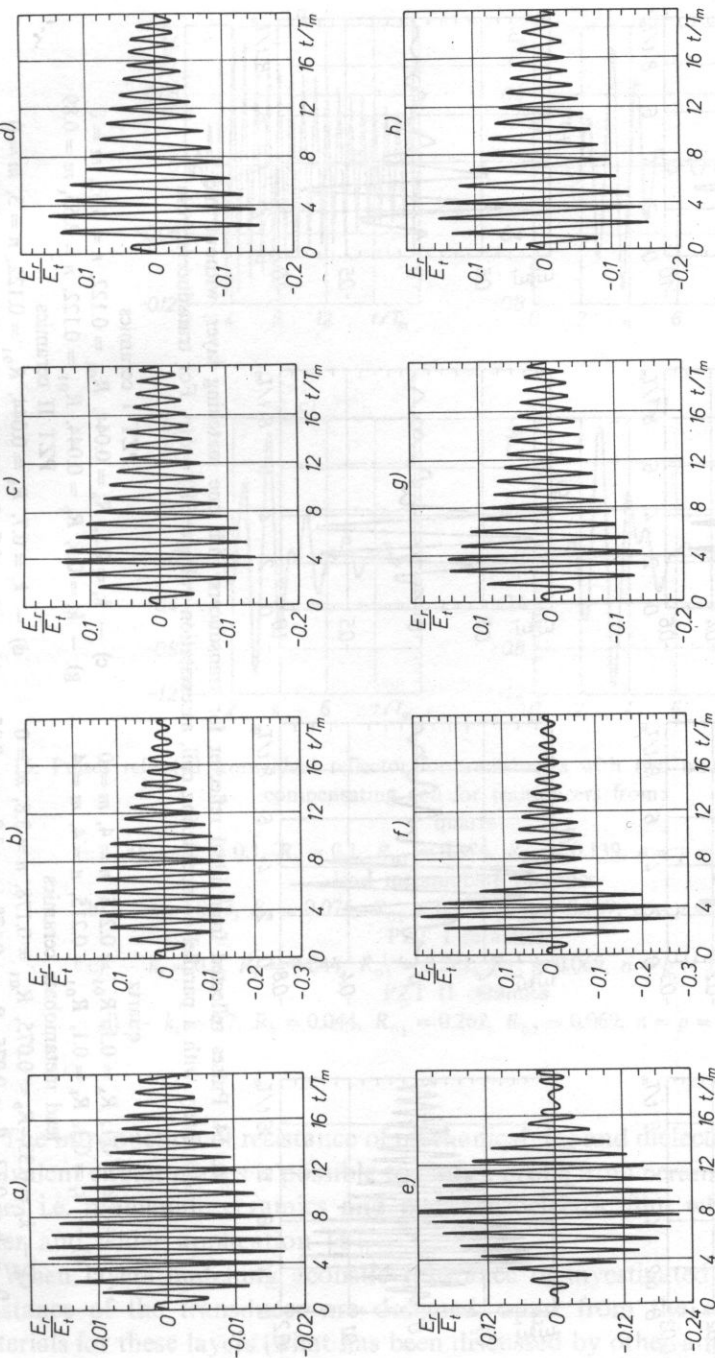


FIG. 13. Pulses reflected from ideal reflector for transducers without matching layers, without and adequately with a parallel compensating coil, at excitation voltage $E(t) = \delta(t)$. For transducers from:

PZT I ceramics transducers

c) - $k_t = 0.5$, $R_b = 0.044$, $m = 0$

g) - $k_t = 0.5$, $R_b = 0.044$, $m = 0.89$

PZT II ceramics transducers

d) - $k_t = 0.7$, $R_b = 0.044$, $m = 0$

h) - $k_t = 0.7$, $R_b = 0.044$, $m = 0.75$

quartz

a) - $k_t = 0.1$, $R_b = 0.1$, $m = 0$

e) - $k_t = 0.1$, $R_b = 0.1$, $m = 1$

lead metaniobate ceramics transducers

b) - $k_t = 0.33$, $R_b = 0.075$, $m = 0$

f) - $k_t = 0.33$, $R_b = 0.075$, $m = 0.95$

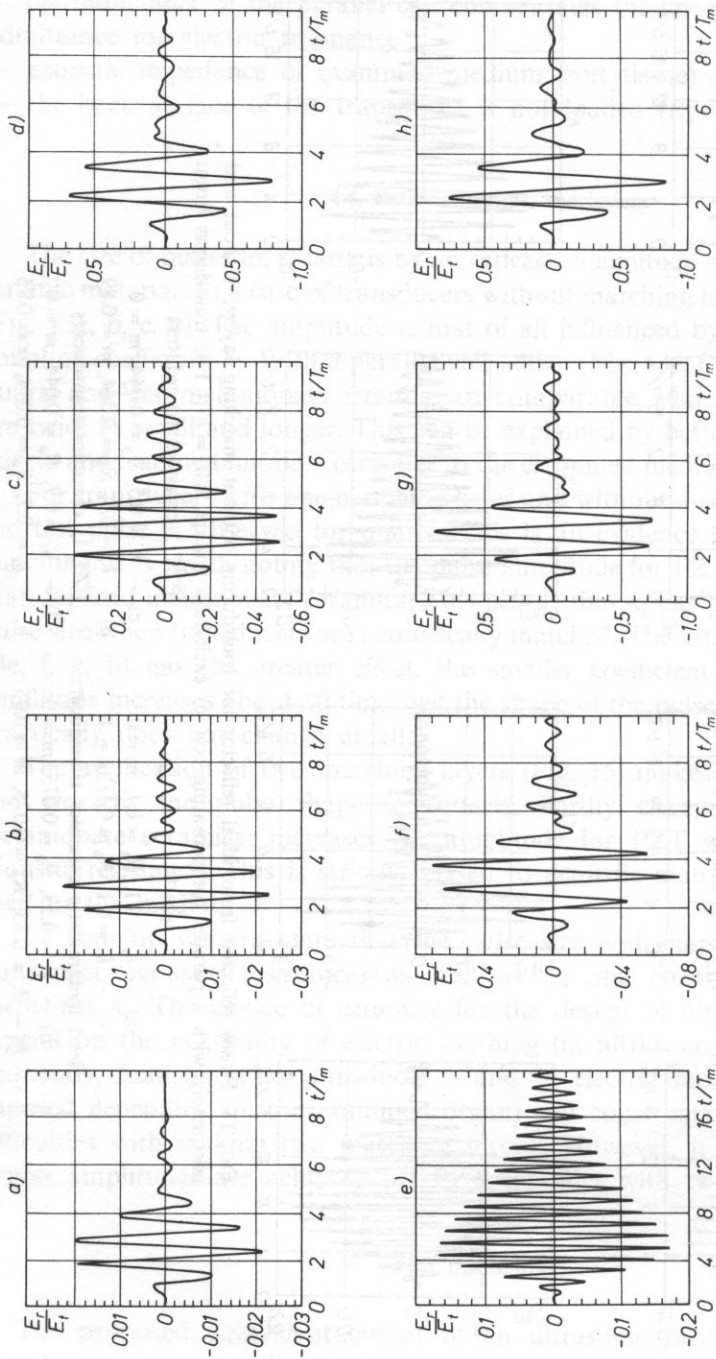


FIG. 14. Pulses reflected from ideal reflector for transducers with one matching layer, without and adequately with a parallel compensating coil, at excitation voltage $E(t) = \delta(t)$. For transducers from:

quartz

a) - $k_t = 0.1$, $R_b = 0.1$, $R_{01} = 0.215$, $n = 4$, $m = 0$

e) - $k_t = 0.1$, $R_b = 0.1$, $R_{01} = 0.215$, $n = 4$, $m = 4$

PZT I ceramics

c) - $k_t = 0.5$, $R_b = 0.044$, $R_{01} = 0.122$, $n = 3.55$, $m = 0$

g) - $k_t = 0.5$, $R_b = 0.044$, $R_{01} = 0.122$, $n = 3.55$, $m = 0.89$

PZT II ceramics

c) - $k_t = 0.33$, $R_b = 0.075$, $R_{01} = 0.178$, $n = 3.8$, $m = 0$

d) - $k_t = 0.33$, $R_b = 0.075$, $R_{01} = 0.178$, $n = 3.8$, $m = 0.95$

d) - $k_t = 0.7$, $R_b = 0.044$, $R_{01} = 0.122$, $n = 3$, $m = 0$

h) - $k_t = 0.7$, $R_b = 0.044$, $R_{01} = 0.122$, $n = 3$, $m = 0.75$

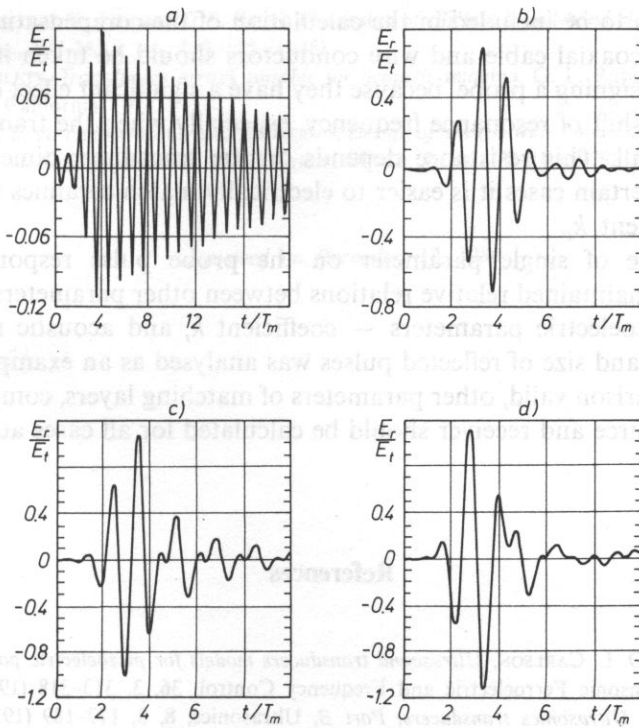


FIG. 15. Pulses reflected from ideal reflector for transducers with two matching layers and parallel compensating coil for transducers from:

- quartz
- a) - $k_t = 0.1$, $R_b = 0.1$, $R_{01} = 0.374$, $R_{02} = 0.139$, $n = p = 4$, $m = 1$
 lead metaniobate ceramics
- b) - $k_t = 0.33$, $R_b = 0.075$, $R_{01} = 0.329$, $R_{02} = 0.109$, $n = p = 3.8$, $m = 0.95$
 PZT I ceramics
- c) - $k_t = 0.5$, $R_b = 0.044$, $R_{01} = 0.262$, $R_{02} = 0.069$, $n = p = 3.55$, $m = 0.89$
 PZT II ceramics
- d) - $k_t = 0.7$, $R_b = 0.044$, $R_{01} = 0.262$, $R_{02} = 0.069$, $n = p = 3$, $m = 0.75$

The introduction of resistance of mechanical, R_m and dielectric losses, R_e , into the equivalent circuit makes it possible to design probe from ceramics with high internal losses i.e. niobianate ceramics and pvdf piezoelectric film which are now finding wider and wider application [6].

When layers matching acoustic resistance of investigated medium to acoustic resistance of the transducer are designed, apart from the selection of adequate materials for these layers (what has been discussed by other authors), the shift of the transducer electric resonance, depending on the electromechanical coupling coefficient k_p , has to be taken into account when the thickness of these layers is determined.

This shift has to be included in the calculation of the compensating system. The influence of the coaxial cable and wire conductors should be taken into account in the process of designing a probe, because they have a significant effect on the value of admittance and shift of resonance frequency, especially when the transducer electric resistance is small. This resistance depends on the transducer dimensions and its parameters. In certain cases it is easier to electrically match ceramics with a smaller, coupling coefficient k_t .

The influence of single parameter on the probe pulse response should be compared with maintained relative relations between other parameters. In this paper influence of piezoelectric parameters — coefficient k_t and acoustic resistance $\rho_p c_p$ — on the shape and size of reflected pulses was analysed as an example. In order to make this comparison valid, other parameters of matching layers, compensatory coil, resistances of source and receiver should be calculated for all cases according to the same formulae.

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In the measurement of actual random vibrations, the observed data often result in a loss of a large portion of information due to the case that the collected observed types of measurement equipments. In this paper, a useful extension of the frequency to explain sound data for an environmental noise at the same level is proposed in an actual case where the wave has a finite range of amplitude frequency as well as is measured through the sound level meter with a limited frequency range. The resultant expression of the probability density function describing the observed data in terms of the sum of finite series type expansion function. This derivation is an L_2 expansion in the orthogonal polynomial in the orthogonal polynomial. The form of probability expansion function is well-known statistical Laguerre polynomial expansion as a special case, and the statistical Laguerre type expansion series type probability is analogous to the special function case. Finally, the validity of the proposed theory has been experimentally confirmed by applying to the actual noise of type of road traffic noise. The statistical Laguerre series expansion shows good agreement with experimentally summed points as compared with other types of statistical case with the.

1. Introduction

It is well known that the Gaussian distribution is of essential importance as a standard type probability distribution expression of random sound or vibration, not only in the statistical test also in the indoor acoustics. Moreover, it is inferior to the Gaussian distribution as a probability distribution for the fluctuation of the sound intensity — for instance in the A-weighted sound data, the Gamma distribution plays an