ARCHIVES OF ACOUSTICS
1, 3-4, 253-270 (1982)

DETECTABILITY OF BLOOD VESSELS BY MEANS OF THE ULTRASONIC ECHO METHOD USING A FOCUSED ULTRASONIC BEAM

LESZEK FILIPCZYŃSKI

Department of Ultrasound, Institute of Fundamental Technological Research, Polish Academy of Sciences
(00-049 Warszawa, ul. Świętokrzyska 21)

The detectability of a small, hypothetical cylinder-shaped blood vessel with a diameter of 0.1 mm has been considered analytically using the ultrasonic echo method. Soft tissues surrounding the vessel have been taken as homogeneous and not causing reflections of ultrasonic waves. They have been ascribed both with bulk and shear elasticity. A beam of longitudinal ultrasonic waves incident on the vessel has been taken in the form of a focused beam at the focus of which the blood vessel has been placed. It has been assumed that the reflection of ultrasonic waves from the blood vessel is caused by the difference between the velocity of waves in the tissue surrounding the vessel and that in blood.

Assuming a frequency of ultrasonic waves of 2.6 MHz, a diameter of the transmit-receive piezoelectric transducer of 2 cm, a focal length of this transducer of 10 or 8 cm, voltage of the transmitter of 250 V and sensitivity of the receiver of 10^{-5}V, the conditions of detectability have been determined.

It has been shown that the signal of an echo from the blood vessel assumed is potentially detectable. Its magnitude depends critically on the distance between the vessel and the surface of the body, resulting from the attenuation of waves in tissues penetrated.

1. Introduction

In an earlier paper [3], devoted to the detectability of blood vessels, the present author assumed that the ultrasonic beam incident on the blood vessel is parallel. The present considerations deal with a focused ultrasonic beam which is used in most modern diagnostic apparatus.

Another change in the assumptions made for the present problem is the taking of a more exact and more complex model of soft tissue in which ultrasonic waves propagate. The present author assumes that this tissue exhibits not only bulk elasticity but also shear elasticity. In the previous work [3] only the bulk elasticity of tissue was assumed.

The problem of parameters characterizing the shear elasticity of soft tissues, however, remains. One of the sources of information on this subject is the paper of FRIZELL and others [6], who measured the characteristic impedance of these tissues for transverse waves and determined on this basis the order of the velocity and absorption of transverse waves in tissues of this type. The present author used these data in obtaining the quantitative results of the analytical expressions derived.

Notation

A, A_z — vector potential of displacement
A_l — attenuation loss
a — radius of blood vessel
a_p — radius of piezoelectric transducer
B_m — constant
b — wave number for blood, dipole moment
c — velocity of longitudinal wave in tissue
c_t — velocity of transverse wave in tissue
c_b — velocity of longitudinal wave in blood
C_m, C_m^* — constants
D_m, D_m^* — constants
f — focal length
$H_m^{(2)}$ — Hankel function of second kind
h — wave number of transverse wave in tissue
J_m — Bessel function
j — $\sqrt{-1}$
k — wave number of longitudinal wave in tissue
M — mass
m — natural number
N_m — Neuman function
N_r — power of wave incident on transducer
N_t — power of wave radiated by transducer
Q_0 — volume velocity of source
p — acoustic pressure
R — reflection loss
r — distance from transducer, coordinate of polar system
S — area
s — current radius on transducer surface
T — transducing loss
t — time
$\boldsymbol{u}, u_r$ — displacement vector, its radial component
u_0, u_1, U_0, U_1 — vibration velocity of sources (instantaneous values and amplitudes)
$\boldsymbol{v}, v_r$ — vector of acoustic velocity, its radial component
w — vibration velocity of transducer surface
x — argument of cylindrical functions
Y — axis of blood vessel
y — coordinate on Y axis
Z_m — cylindrical function

$a_m(J)$, $a_m(H)$ — auxiliary constants
β_m — auxiliary constant
γ_m — auxiliary constant
$\delta_m(J)$, $\delta_m(H)$ — auxiliary constants
ε — radius tending to zero
η — displacement potential in blood
θ — azimuth
Λ — wavelength in tissue
λ — Lame constant in tissue
λ_b — Lame constant in blood
μ — Lame constant in tissue
ϱ — density of tissue
ϱ_b — density of blood
σ_{rr} — normal stresses
Φ — displacement potential in tissue
φ — velocity potential in tissue
ψ — angle
ω — angular velocity

2. Assumptions of analysis

It can be assumed that a cylinder-shaped vessel with the radius $a = 0.1$ mm is placed in the focus of an ultrasonic beam generated by a bowl-shaped focusing piezoelectric transducer (Fig. 1). The diameter of the transducer is $2a_p = 2$ cm

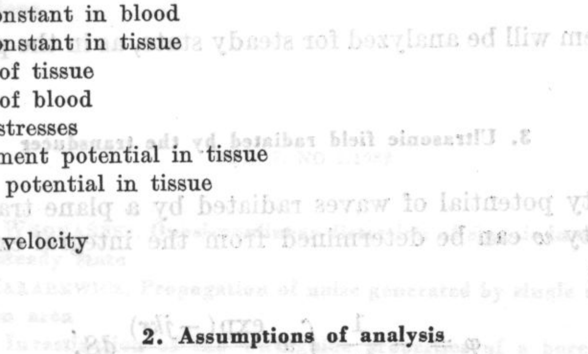

Fig. 1. A scattered reflection of ultrasonic waves from a small blood vessel
T – transmitter, R – receiver, P – ultrasonic probe with piezoelectric transducer, v – small blood vessel, Φ_i – incident wave, Φ_r – reflected wave, ST – soft tissue, f – focal length of the transducer

and its focal length is $f = 10$ cm. An ultrasonic wave of a frequency of 2.6 MHz (wavelength $\Lambda = 0.63$ mm) is incident on the blood vessel, perpendicularly to its axis. The soft tissue surrounding the vessel and the walls of the vessel are assumed to be homogeneous and to have the same characteristic acoustic impedance. In this case it can be assumed that the reflection of the ultrasonic wave from the blood vessel is caused only by a difference between the velocity of the wave in the tissue surrounding the vessel and that in blood. The densities of the tissue and blood are assumed to be the same. The velocity of longitudinal ultrasonic waves in muscle tissue is assumed to be $c = 1.63$ km/s (uterus muscle), while that in blood is taken as $c_b = 1.57$ km/s [7].

The velocity of transverse waves in muscle tissue is known only in terms of the order of magnitude. FRIZELL and others [6] determined the velocity range of these waves to be 9-100 m/s; it is then possible to assume for calculations the velocity of transverse waves $c_t = 65$ m/s which falls within the range mentioned above.

This problem will be analyzed for steady state, as in the previous paper [3].

3. Ultrasonic field radiated by the transducer

The velocity potential of waves radiated by a plane transducer vibrating with the velocity w can be determined from the integral expression given by RAYLEIGH [10]

$$\varphi = \frac{1}{2\pi}\int_S w\frac{\exp(-jkr)}{r}dS, \tag{1}$$

where S is the vibrating area, $k = 2\pi/\Lambda$ and r is the distance from the transducer. Expression (1) can be regarded as a quantitative representation of the Huygens principle. The acoustic velocity v, the velocity potential φ and the acoustic pressure ϱ are connected by the relations

$$v = -\operatorname{grad}\varphi, \tag{2}$$

$$p = \varrho\frac{\partial\varphi}{\partial t}. \tag{3}$$

Expression (1) is satisfied exactly only for plane vibrating surfaces; it can also be used, however, in the case of weakly focusing transducers [9] when the following conditions are satisfied,

$$a_p \ll f, \tag{4}$$

$$a_p \gg \Lambda, \tag{5}$$

where a_p is the radius of the transducer and f is its focal length.

From expressions (1) and (3) it is possible to determine the acoustic pressure at a point of the field P (Fig. 2) lying on the Y axis of the blood vessel. Let us consider the surface vibrating element dS at the point Q on the surface of the piezoelectric transducer. The distance $r = QP$ can be expressed in the following way ($s \ll f$),

$$r = f\left(1+\frac{y^2-2sy\cos\psi}{f^2}\right)^{1/2} \approx f\left(1+\frac{y^2-2sy\cos\psi}{2f^2}\right), \tag{6}$$

where $y = GP$, s is the distance between the point Q and the X axis and ψ is the angle between the Y axis and the plane 0' QG. The error resulting from simplification (6) is equal to [5]

$$\Delta r = \frac{1}{8f^3}(y^2 - 2\,sy\cos\psi)^2 \tag{7}$$

and in the worst case (for $s = a_p$ and $\cos\psi = -1$) it is smaller by a factor of about 60 than the wavelength. The distance r in the denominator of expression (1)

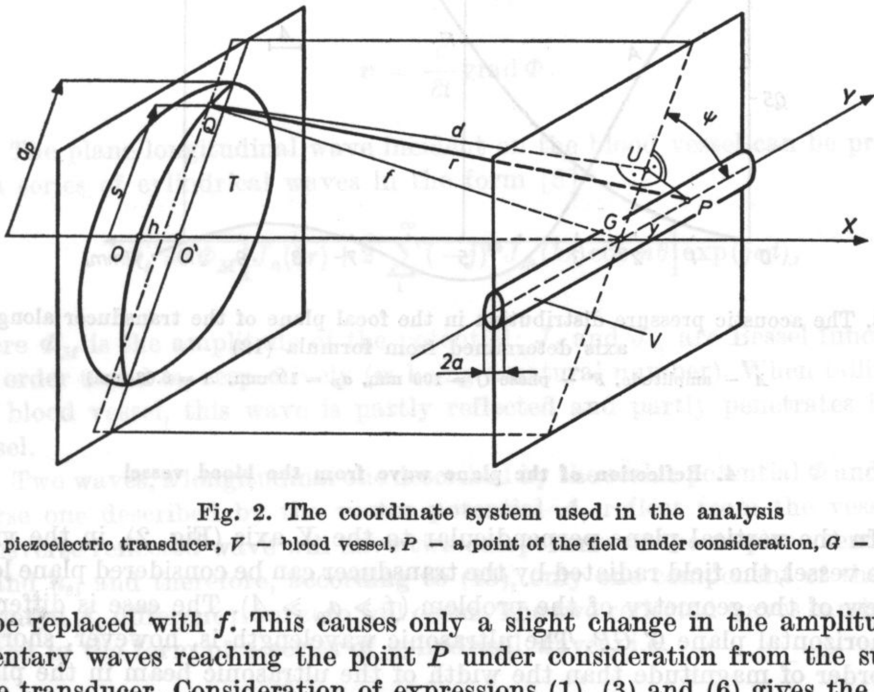

Fig. 2. The coordinate system used in the analysis

T — piezoelectric transducer, *V* — blood vessel, *P* — a point of the field under consideration, *G* — focus

can be replaced with f. This causes only a slight change in the amplitude of elementary waves reaching the point P under consideration from the surface of the transducer. Consideration of expressions (1), (3) and (6) gives the value of acoustic pressure

$$p = \frac{jk\varrho cw}{2\pi f}\exp\left\{j\left[\omega t - kf\left(1 + \frac{y^2}{2f^2}\right)\right]\right\}\int\limits_0^{a_p}\int\limits_0^{2\pi}\exp\left(\frac{j\,ksy\cos\psi}{f}\right)d\psi s ds. \tag{8}$$

Application of the known properties of Bessel functions

$$J_0(x) = \frac{1}{\pi}\int\limits_0^{\pi}\exp(jx\cos\psi)\,d\psi, \tag{9}$$

$$\int xJ_0(x)/dx = xJ_1(x), \tag{10}$$

$$J_1(0) = 0, \tag{11}$$

yields, from expression (8), the following acoustic pressure distribution in the focal plane of the transducer, along the Y axis,

$$p = \frac{jk\varrho c w a_p^2}{2f} \exp\left\{j\left[\omega t - kf\left(1 + \frac{y^2}{2f^2}\right)\right]\right\} \frac{2J_1(ka_p y/f)}{ka_p y/f}. \tag{12}$$

This distribution is shown in Fig. 3 for the system assumed here.

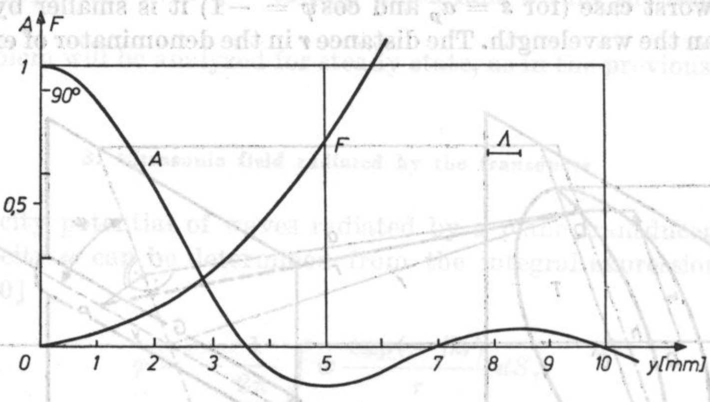

Fig. 3. The acoustic pressure distribution in the focal plane of the transducer along the Y axis determined from formula (12)

A – amplitude, F – phase ($f = 100$ mm, $a_p = 10$ mm, $\Lambda = 0.63$ mm)

4. Reflection of the plane wave from the blood vessel

In the vertical plane perpendicular to the Y axis (Fig. 2), in the vicinity of the vessel, the field radiated by the transducer can be considered plane locally, in view of the geometry of the problem ($f \gg a_p \gg \Lambda$). The case is different in the horizontal plane $0'$ GP. The ultrasonic wavelength is, however, shorter by one order of magnitude than the width of the ultrasonic beam in the plane of the focus (Fig. 3). In view of this, it can be assumed with some approximation that the reflection of an ultrasonic wave in each cross-section of the vessel will be independent of the adjacent cross-sections. As a consequence, the partial contributions, obtained independently for the reflection of the wave in each plane section of the vessel, can be gathered, giving the solution of the problem of reflection which takes into account the distribution of the wave incident along the Y axis.

Analysis can be carried out in a plane system of polar coordinates where r is the radius and θ is the azimuth. The axis of the coordinate system is coaxial with the axis of the blood vessel. The present analysis will consider both the bulk and shear clasticity of the soft tissue in which the wave radiated by the transducer propagates. In view of this, it is more covenient to replace the velocity potential (employed in acoustics) with scalar and vector displacement potentials (used in elasticity theory). Thus Φ denotes here the scalar potential of the displacement vector $\boldsymbol{u}$, $\boldsymbol{A}$ the vector potential of the displacement vector. The following relation then occurs,

$$\boldsymbol{u} = \operatorname{grad} \Phi + \operatorname{rot} \boldsymbol{A}. \tag{13}$$

It should be noted that in the case of the scalar displacement potential, relations (2) and (3) take other form, i.e.

$$p = -\varrho \frac{\partial^2 \Phi}{\partial t^2}, \tag{13a}$$

$$\boldsymbol{v} = \frac{\partial}{\partial t} \operatorname{grad} \Phi. \tag{13b}$$

The plane longitudinal wave incident on the blood vessel can be presented as a series of cylindrical waves in the form [8]

$$\Phi_i = \Phi_M \Big[J_0(kr) + 2 \sum_1^{\infty} (-j)^m J_m(kr) \cos m\theta \Big] \exp(j\omega t), \tag{14}$$

where Φ_M is the amplitude of the potential; J_0 and J_m are Bessel functions of the order 0 and m, respectively (m being a natural number). When falling onto the blood vessel, this wave is partly reflected and partly penetrates into the vessel.

Two waves, a longitudinal one described by the scalar potential Φ and a transverse one described by the vector potential $\boldsymbol{A}$, reflect from the vessel. The transverse reflected wave will have two components of the displacement vector, u_r and u_θ, and therefore, according to (13), only one component of the vector potential A_z different from zero will occur. The two reflected waves can be represented in the form of a series of cylindrical waves, i.e.

$$\Phi_r = \sum_0^{\infty} D_m H_m^{(2)}(kr) \cos(m\theta) \exp(j\omega t), \tag{15}$$

$$A_z = \sum_0^{\infty} C_m H_m^{(2)}(hr) \sin(m\theta) \exp(j\omega t), \tag{16}$$

where $h = \omega / c_t$ is the wave number for the transverse wave and $H_m^{(2)}$ is a Hankel function of the 2nd kind.

The function $\cos\theta$ occurs in formula (15), since the longitudinal reflected wave is symmetrical with respect to the incidence direction of the wave ($\theta = 180°$), whereas $\sin\theta$ occurs in formula (16), since the transverse wave is antisymmetrical. This results from the geometry of the problem.

Only the longitudinal wave penetrates into the blood vessel. This wave can be assumed in the form

$$\eta = \sum_{0}^{\infty} B_m J_m(br)\cos(m\theta)\exp(j\omega t), \tag{17}$$

where η is the scalar displacement potential; $b = \omega/c_b$, c_b is the velocity of the wave in blood.

The constants D_m, C_m and B_m in relations (15)-(17) can be determined from three boundary conditions which must be satisfied on the boundary of the vessel $r = a$.

The first condition is the equality of the normal stresses σ_{rr} in tissue and the acoustic pressure p in blood on the boundary of the vessel. It takes the form

$$\sigma_{rr} = \lambda \operatorname{div} \boldsymbol{u} + 2\mu \frac{\partial u_r}{\partial r} = -p, \tag{18}$$

where λ and μ are Lame constants.

The minus sign on the right side of condition (18) results from the fact that in mechanics tensile stresses are regarded as positive, while in acoustics the corresponding pressures are considered to be negative.

Consideration of (13) in (18); of the formulae

$$\operatorname{div}\operatorname{rot} = 0, \tag{19a}$$

$$\operatorname{div}\operatorname{grad} = \nabla^2; \tag{19b}$$

of the wave equations which are satisfied by the displacement potentials

$$\nabla^2\Phi + k^2\Phi = 0, \tag{20a}$$

$$\nabla^2 A_z + h^2 A_z = 0, \tag{20b}$$

$$\nabla^2\eta + b^2\eta = 0, \tag{20c}$$

where

$$k = \frac{\omega}{c} = \sqrt{\frac{\varrho\omega^2}{\lambda + 2\mu}}, \tag{21a}$$

$$h = \frac{\omega}{c_t} = \sqrt{\frac{\varrho\omega^2}{\mu}}, \tag{21b}$$

$$b = \frac{\omega}{c_b} = \sqrt{\frac{\varrho_b\omega^2}{\lambda_b}}, \tag{21c}$$

(ϱ_b being the density of blood, λ_b a Lame constant in blood), and also of the relation between the sound pressure p and the displacement potential

$$p = -\varrho\frac{\partial^2\eta}{\partial t^2}, \tag{22}$$

gives the boundary c ndition (18) for $r = a$, in the form

$$\Phi\left(\frac{2k^2}{h^2}-1\right)+\frac{2}{h^2}\,\Phi''+\frac{2}{h^2}\left(\frac{1}{r}\,A_\theta\right)'=-\frac{\varrho_b}{\varrho}\,\eta. \tag{23}$$

In order to simplify notation the index z was dropped in the component of the vector potential A_z. The dash in the potentials denotes differentiation with respect to r, while the index θ denotes differentiation with respect to θ.

The second boundary condition involves the disappearance of the tangential stresses on the boundary of the vessel ($r = a$). This condition has the form [2]

$$\frac{2}{r}\,\Phi_\theta-\frac{2}{r^2}\,\Phi_\theta-A''+\frac{1}{r}\,A'+\frac{1}{r^2}\,A_{\theta\theta}=0. \tag{24}$$

The third boundary condition requires the equality of the normal displacements on the boundary of the vessel ($r = a$). Consideration of (13) gives

$$\Phi'+\frac{1}{r}\,A_\theta=\eta'. \tag{25}$$

Insertion into the above conditions of the sum $\Phi = \Phi_i + \Phi_r$ whose components are expressed by expressions (14) and (15) and supstitution of potentials (16) and (17) give the possibility of determining the constants D_m and C_m of interest here which describe the waves reflected from the blood vessel. In view of the orthogonality of the sine and cosine functions, these constants can be determined independently, successively for each m [8]. In calculating the derivatives of cylindrical functions it is possible to use the formulae

$$\frac{dZ_m(kr)}{dr}=k\left[\frac{m}{kr}\,Z_m(kr)-Z_{m+1}(kr)\right], \tag{26}$$

$$\frac{d^2Z_m(kr)}{dr^2}=k^2\left\{\left[\frac{m(m-1)}{(kr)^2}-1\right]Z_m(kr)+\frac{1}{kr}\,Z_{m+1}(kr)\right\}. \tag{27}$$

Thus, for $m = 0$

$$D_0=\frac{\dfrac{2}{h^2}\,J_0''(kr)J_0'(br)-\left(\dfrac{2k^2}{h^2}-1\right)J_0(kr)J_0'(br)-\dfrac{\varrho_b}{\varrho}\,J_0'(kr)J_0(br)}{H_0^{(2)}(kr)\left(\dfrac{2k^2}{h^2}-1\right)J_0'(br)+\dfrac{2}{h^2}\,H_0^{(2)''}(kr)J_0'(br)+\dfrac{\varrho_b}{\varrho}\,H_0^{(2)'}(kr)J_0(br)}\,\Phi_M; \tag{28}$$

while the constant C_0 is equal to zero.

For $m \geqslant 1$, however, the following expression can be obtained,

$$D_m=\frac{-a_m(J)\,\beta_m-\gamma_m\delta_m(J)}{a_m(H)\,\beta_m+\gamma_m\delta_m(H)}\,\Phi_m, \tag{29}$$

where

$$\alpha_m(J) = \frac{2m}{r}\left[J'_m(kr) - \frac{1}{r}J_m(kr)\right], \tag{30a}$$

$$\alpha_m(H) = \frac{2m}{r}\left[H_m^{(2)'}(kr) - \frac{1}{r}H_m^{(2)}(kr)\right], \tag{30b}$$

$$\beta_m = \frac{2m}{h^2}\left[\frac{H_m^{(2)}(hr)}{r}\right]' + \frac{m}{r}\frac{\varrho_b}{\varrho}J_m(br)\frac{H_m^{(2)}(hr)}{J'_m(br)}, \tag{30c}$$

$$\gamma_m = \left[\frac{1}{r}H_m^{(2)'}(hr) - H_m^{(2)''}(hr) - \frac{m^2}{r^2}H_m^{(2)}(hr)\right], \tag{30d}$$

$$\delta_m(J) = \left(\frac{2k^2}{h^2} - 1\right)J_m(kr) + \frac{2}{h^2}J''_m(kr) + \frac{\varrho_b}{\varrho}J_m(br)\frac{J'_m(kr)}{J'_m(br)}, \tag{30e}$$

$$\delta_m(H) = \left(\frac{2k^2}{h^2} - 1\right)H_m^{(2)}(kr) + \frac{2}{h^2}H_m^{(2)''}(kr) + \frac{\varrho_b}{\varrho}J_m(br)\frac{H_m^{(2)'}(kr)}{J'_m(br)}, \tag{30f}$$

and $r = a$.

The constant C_m which occurs in expression (16) describing the transverse reflected waves can be determined from the second boundary condition (24). The following relation can then be obtained

$$C_m = \frac{\dfrac{2m}{r}J'_m(kr) - \dfrac{2m}{r^2}J_m(kr) - D_m\left[\dfrac{2m}{r^2}H_m^{(2)}(kr) - \dfrac{2m}{r}H_m^{(2)'}(kr)\right]\dfrac{1}{\Phi_M}}{\dfrac{1}{r}H_m^{(2)'}(hr) - H_m^{(2)''}(hr) - \dfrac{m^2}{r^2}H_m^{(2)}(hr)}\Phi_M, \tag{31}$$

$$r = a.$$

For the wave velocities in soft tissue and blood assumed above, the following wave numbers can be obtained: $k = 10\ \text{mm}^{-1}$, $h = 250\ \text{mm}^{-1}$ and $b = 10.4\ \text{mm}^{-1}$. When inserted into relations (30), (31) and (32), these wave numbers permit the determination of the constants D_m and C_m which are shown in Table I.

It follows therefore that the constants D_m, and accordingly the magnitudes of the longitudinal reflected waves, decrease rapidly with increasing the wave order m, whereas the constants C_m corresponding to the transverse waves show an oscillatory character, which is understandable in view of the large value of the argument $ha = 25$.

However, in view of very large attenuation, transverse waves exist only in the closest vicinity of the blood vessel. It follows from the measurements of FRIZELL and others [6] that the attenuation of these waves in soft tissues falls within the range (2-30)·10^3 cm^{-1}. Accordingly, the further analysis aimed at the determination of the detectability of a blood vessel will consider only longitudinal waves and neglect the transverse waves which occur for the phenomenon of reflection. Moreover, only the first two orders of reflected waves ($m = 0.1$) will be considered. In view of the rapid decrease in the constants D_m with increasing m (see Table 1), the higher orders do not make a noteworthy contribution to the calculation of the amplitude of the longitudinal wave reflected from the blood vessel.

Table 1. The constants D_m^* and C_m^* determined from formulae (28), (29) and (31)

m	$D_m^* = D_m/\Phi_M$	$C_m^* = C_m/\Phi_M$
0	$0.0476e^{-j93°}$	0
1	$0.0068e^{-j90°}$	$0.0025e^{j43°}$
2	$0.0008e^{j176°}$	$0.0045e^{-j52°}$
3	$0.00002e^{j40°}$	$0.0023e^{+j52°}$

Insertion into formula (13a) of expression (15) and of the constants D_0 and D_1 gives the acoustic pressure of the wave reflected from the blood vessel in the form

$$p = \omega^2 \varrho [D_0 H_0^{(2)}(kr) + D_1 H_1^{(2)}(kr)\cos\theta]\exp(j\omega t). \tag{32}$$

The acoustic velocity of the reflected wave can be determined in a similar way from expressions (13*b*) and (15),

$$v_r = j\omega\left\{-D_0 k H_1^{(2)}(kr) + D_1\left[\frac{1}{r}H_1^{(2)}(kr) - kH_2^{(2)}(kr)\right]\cos\theta\right\}\exp(j\omega t). \tag{33}$$

4. Pulsating and oscillating equivalent sources of the wave reflected from the vessel

The wave reflected from the blood vessel can be substituted for by equivalent waves radiated by pulsating and oscillating sources placed on the Y axis of the vessel. Expression (32) shows that these sources will be pulsating (monopoles) and oscillating (dipoles). It can be assumed that these sources have the shape of a sphere with the radius ε tending to zero. These spheres vibrate so that points of their surfaces have the velocity

$$u_0 = U_0 \exp(j\omega t), \tag{34a}$$

$$u_1 = U_1 \cos\theta \exp(j\omega t), \tag{34b}$$

where U_0 and U_1 are the amplitudes of the velocities.

These velocities can be compared with the velocities defined by relation (33) for the small arguments $kr = k\varepsilon$. The Hankel functions which occur in this relation can be represented in the form

$$H_m^{(2)}(x) = J_m(x) - jN_m(x), \tag{35}$$

while for $x \to 0$ and $m > 0$,

$$J_m(x) \to 0, \tag{36a}$$

$$N_m(x) \to -\frac{(m-1)!}{\pi}\left(\frac{2}{x}\right)^m. \tag{36b}$$

It follows therefore that in the present case ($\varepsilon \to 0$)

$$H_1^{(2)}(k\varepsilon) = -\frac{2j}{\pi k\varepsilon}, \tag{37a}$$

$$H_2^{(2)}(k\varepsilon) = \frac{-4j}{\pi k^2\varepsilon^2}. \tag{37b}$$

Substitution of (37a) and (37b) into (33) and equating the sum (34a), (34b) to expression (33) give

$$U_0 + U_1 \cos\theta = -\omega\frac{2D_0}{\pi\varepsilon} - \omega\frac{2D_1}{\pi k\varepsilon^2}\cos\theta. \tag{38}$$

Since this equation should be satisfied for all values of the angle θ,

$$U_0 = -\omega\frac{2D_0}{\pi\varepsilon}, \tag{39a}$$

$$U_1 = -\omega\frac{2D_1}{\pi k\varepsilon^2}. \tag{39b}$$

It is known [11], on the other hand, that a pulsating source radiates a sound pressure wave in the form

$$p_0 = \frac{j\omega\varrho Q_0}{4\pi r}\exp[j(\omega t - kr)], \tag{40}$$

where Q_0 is the amplitude of the volume velocity of the source when its dimensions tend to zero and r is the distance from the source.

It is now possible to form an equivalent cylindrical source (Fig. 4) of the same volume velocity Q_p, whose dimensions are very small compared to the wavelength. The change in the shape of the source from spherical to cylindrical shape has, in view of the relation $\Lambda \gg \varepsilon \to 0$, no significance. It is therefore possible to write (see Fig. 4)

$$dQ_0 = 2\pi\varepsilon U_0 dy. \tag{41}$$

In turn, the acoustic pressure of the wave radiated by an oscillating source at a long distance from the source has the form [11]

$$p_1 = jk\frac{b}{r}\cos\theta\exp[j(\omega t - kr)], \tag{42}$$

where b denotes a dipole moment equal to

$$b = \frac{j\omega\varrho\varepsilon^3\exp(jk\varepsilon)}{(2-k^2\varepsilon^2)+j2k\varepsilon}U_1 \underset{\varepsilon\to 0}{=} j\frac{\omega\varrho\varepsilon^3}{2}U_1. \tag{43}$$

Introducing the mass of an oscillating sphere, $M = \varrho 4\pi\varepsilon^3/3$, it is possible to write

$$b = j\frac{3\omega M U_1}{8\pi}. \tag{44}$$

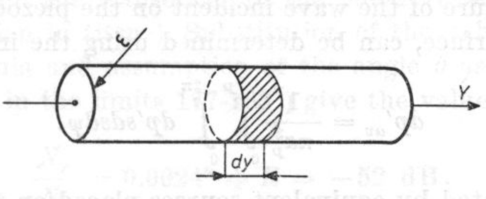

Fig. 4. An equivalent cylindrical wave source with the radius ε and the length dy

Expressions (42) and (44) show that equivalent oscillating sources must have the same momentum with constant frequency. It is therefore possible to form a substitute equivalent cylindrical source. In view of the relation $\Lambda \gg \varepsilon \to 0$, the shape of the source is insignificant. The momentum of an equivalent oscillating source can be written in the form

$$dMU_1 = \varrho\pi\varepsilon^2 U_1 dy. \tag{45}$$

Comparison of expressions (40), (41) and (39a) and (42), (44), (45) and (39b) gives the acoustic pressure of the wave radiated by the two equivalent cylindrical sources considered above, with the length dy, i.e.

$$dp' = -j\left(\frac{\omega^2\varrho D_0}{\pi r} + \frac{3}{4}\frac{j\omega^2\varrho D_1}{\pi r}\cos\theta\right)\exp[j(\omega t - kr)]dy. \tag{46}$$

Considering relations (13a) and (12), it is finally possible to write the expression for the sound pressure of the wave radiated by the element dy placed on the Y axis of a blood vessel. This pressure is equal to the pressure of the wave reflected from the vessel

$$dp' = \left(\frac{D_0^*}{\pi r} + \frac{3}{4}\frac{jD_1^*}{\pi r}\cos\theta\right)\frac{k\varrho c w a_p^2}{2f}\exp\left\{j\left[\omega t - kf\left(1+\frac{y^2}{2f^2}\right) - kr\right]\right\}\times \times\frac{2J_1(ka_p y/f)}{ka_p y/f}\,dy, \qquad (47)$$

where

$$D_0^* = D_0/\Phi_M, \qquad (47a)$$

$$D_1^* = D_1/\Phi_M. \qquad (47b)$$

6. Ultrasonic waves incident on the transducer

The coordinate system now can be changed by setting its centre at the point G (Fig. 2) and reversing the sense of the X axis.

The sound pressure of the wave incident on the piezoelectric transducer T, averaged over its surface, can be determined using the integral

$$dp'_{av} = \frac{1}{\pi a_p^2}\int_0^{a_p}\int_0^{2\pi} dp' s\,ds\,d\psi. \qquad (48)$$

The waves radiated by equivalent sources placed on the axis of the vessel penetrate obliquely the wall of the vessel, whereas the solution obtained in chapter 3 applies to a plane problem and therefore can be used only for a perpendicular penetration of the wave through the walls of the vessel. The resulting differences in phase and amplitude are so small as to be neglected, which follows from the geometry of the problem assumed ($f \gg a_p$ and $\Lambda \gg a$).

In formula (47) r can be expressed by s and ψ (see Fig. 2). Subsequently the expression for dp' thus changed can be inserted into formula (48) and integration carried out in this formula with respect to the variables s and ψ in the way used in chapter 2. This gives the mean acoustic pressure caused on the surface of the transducer by the wave radiated by cylindrical sources of length dy placed on the Y axis of the blood vessel.

Carrying out, in turn, further integration with respect to the variable y within the limits $\pm\infty$, all the contributions of the elementary waves radiated by the equivalent sources can be gathered. This integration leads to the final result in the form:

$$p'_{av} = \left(D_0^* + \frac{3}{4}jD_1^*\cos\theta\right)\frac{k\varrho c w a_p^2}{2\pi f^2}\exp\{j(\omega t - 2kf)\}\times \times\int_{-\infty}^{+\infty}\left[\frac{2J_1(ka_p y/f)}{ka_p y/f}\right]^2\exp\left(\frac{-jky^2}{f}\right)dy. \qquad (49)$$

In formula (20) given by the present autor in a former paper [4] a mistake was made and π^3 should be replaced with 2π, as it is here in equation (49).

The ratio of the power of the wave incident on the transducer N_r to the power of the wave radiated by this transducer N_t will be determined. In view of the geometry of the system assumed, sufficient approximation is provided in this case by the formulae for a plane wave. Thus

$$\frac{N_r}{N_t} \cong \frac{|p'_{av}|^2 \pi a_p^2}{2\varrho c} : \frac{w^2 \varrho c \pi a_p^2}{2} = \left[\frac{|p'_{av}|}{w\varrho c}\right]^2 \tag{50}$$

or, after substitution of the value p'_{av} from expression (49), finally,

$$\frac{N_r}{N_t} = \left|\left[D_0^* + j\frac{3}{4}D_1^*\cos\theta\right]\frac{ka_p^2}{2\pi f^2}\int_{-\infty}^{+\infty}\left[\frac{2J_1(ka_py/f)}{ka_py/f}\right]^2 \exp(-jky^2/f)dy\right|^2. \tag{51}$$

The above formula permits the calculation of signal losses of an ultrasonic wave on its path from the transducer to the blood vessel and back (without considering attenuation in tissue). Substitution of the values a_p, f, k and $D_{0,1}^*$ into the above formula and assumption of the angle θ as 180° approximately (in practice it varies in the limits 177-183°) give the value

$$\frac{N_r}{N_t} = 0.0024^2 \doteq R = -52 \text{ dB}. \tag{52}$$

R represents here the loss in the wave signal radiated, which occurs as a result of that the wave partly goes round the vessel, partly penetrates into it and of that the reflected wave diverges so that only a slight part of the power of the wave returns to the transducer. The quantity R can be called the reflection loss.

7. Detectability of a blood vessel. Discussion

Assumption of an output voltage of the transducer of 250 V and a typical sensitivity of the ultrasonograph receiver of 10^{-5} V gives a ratio of these quantities equal to $W = 2.5\cdot 10^7 \doteq 148$ dB. The loss caused by the transducing of ultrasonic electrical impulses was assumed as $T = -15$ dB.

Fig. 5 shows the signal level generated by the transmitter and its transducing loss T, the reflection loss R and the attenuation loss A_l in the tissues penetrated. The present hypothetical vessel can be detected at a distance of 10 cm from the surface of the body with attenuation in tissue of 1.8 dB/cm (obstetrics [1]). In the case of attenuation of 3.7 dB (muscle tissue, perpendicular to fibres [7]) the signal will only be stronger by 7 dB than the noise level and it will be more difficult to detect the vessel. When the focal length f is shortened from 10 to 8 cm (dynamic focussing) and the blood vessel is placed there, the reflection loss R decreases by only about 1 dB, while the attenuation loss A_l decreases by as much as 16 dB. The detectability of a blood vessel would thus increase greatly.

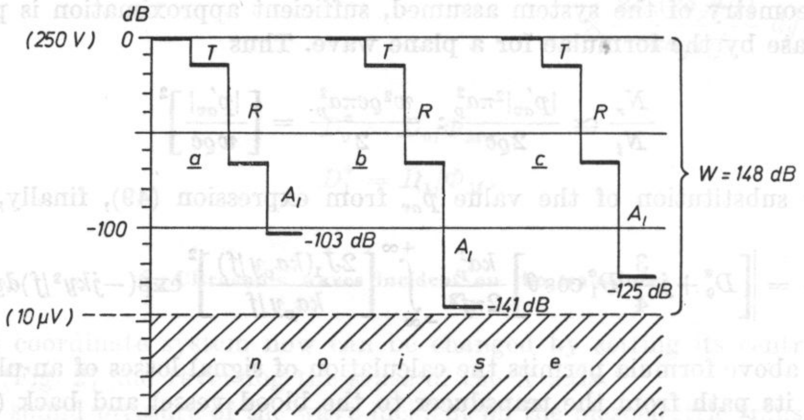

Fig. 5. The signal losses in the detection of a cylindrical blood vessel with a diameter of 0.1 mm by the ultrasonic echo method

T – transducing loss, R – reflection loss, A_l – attenuation loss in tissues penetrated, W – transmitter voltage to receiver voltage sensitivity ratio

a – distance 10 cm, attenuation in tissue 1.8 dB/cm; b – distance 10 cm, attenuation in tissue 3.7 dB/cm; c – distance 8 cm, attenuation in tissue 3.7 dB/cm

It is important to note that the tissues penetrated are inhomogeneous. A number of echoes result from reflections from small anatomical structures (muscle fibres, arterioles, capillaries). In view of this, the signal received from a given blood vessel can, nevertheless, remain undetected amoung other signals.

8. Conclusions

The analysis performed has confirmed the previous conclusions of the author [3] regarding the detectability of small blood vessels. The signals received from a very small blood vessel with a radius of 0.1 placed at a distance of 10 cm from the transducer give potentially detectable signals at a frequency of 2.5 MHz.

The substitution of a focussed ultrasonic beam for a parallel one and the consideration of the shear elasticity of soft tissues, in addition to their bulk elasticity, introduced quantitative changes in the detectability of blood vessels. In the present investigation the reflection loss R was -52 dB, while in the previous study their value was -67 dB. In this case the signals received from the vessel under consideration are stronger by 15 dB. The strength of these signals depends critically on the distance between the vessel and the surface of the body, in view of attenuation loss in the tissues penetrated.

The quantitative results obtained here are estimated only, mainly because of the lack of data describing the acoustic parameters of tissues (e.g. the walls of the vessel). In view of this, the difference between the propagation velocities of waves in the tissue surrounding the vessel and in blood has been assumed as the reason for the reflection of ultrasonic waves from the vessel. It is necessary to note, however, that the conditions assumed here are less favourable for the reflection of ultrasonic waves from a blood vessel than in practice.

References

[1] J. Etienne, L. Filipczyński, A. Firek, J. Groniowski, J. Kretowicz, G. Łypacewicz, J. Sałkowski, *Determination of ultrasonic focused beams used in ultrasonography in the case of gravid uterus,* Ultrasound in Medicine and Biology, **2**, 119-122 (1976).

[2] L. Filipczyński. *Reflection of plane elastic waves from a cylinder with a free surface,* Proc. II Conference on Ultrasonic Technique, PWN, Warsaw 1957, pp. 21-27.

[3] L. Filipczyński, *Detectability of small blood vessels and flat boundaries of soft tissues in the ultrasonic pulse echo method,* Archives of Acoustics, **6**, **1**, 45-62 (1981).

[4] L. Filipczyński, *Detectability of blood vessels with ultrasonic echo method: Physical considerations,* Proc 4-th European Congress on Ultrasonics in Medicine, Ed. A. Kurjak, A. Kratochvil, Dubrovnik 1981, Excerpta Medica, Amsterdam, 81-85.

[5] L. Filipczyński, J. Etienne, *Theoretical study and experiments on spherical focusing transducers with Gaussian surface velocity distribution,* Acustica, **28**, 121-128 (1973).

[6] L. A. Frizell, E. L. Carstensen, J. F. Dyro, *Shear properties of mammalian tissues at low megaherz frequencies,* JASA, **60**, 1409-1411 (1976).

[7] A. S. Goss, R. L. Johnston, R. Dunn, *Compilation of empirical ultrasonic properties of mamalian tissues.* II, JASA, **68**, 93-107 (1980).

[8] P. Morse, *Sound and vibration,* McGraw Hill, New York 1948.

[9] H. T. O'Neil, *Theory of focusing radiators,* JASA, **21**, 516 (1943).

[10] Lord Rayleigh, *Theory of sound,* London 1945.

[11] R. N. Rshevkin, *Kurs lektsij po teorii zwuka,* Moscow 1960.

Received on July 7, 1981; revised on January 19, 1982.