Blood Scattering Model for Pulsed Doppler

Piotr KARWAT, Andrzej NOWICKI, Marcin LEWANDOWSKI

Institute of Fundamental Technological Research
Polish Academy of Sciences
Pawińskiego 5B, 02-106 Warszawa, Poland
e-mail: pkarwat@ippt.gov.pl

(received July 10, 2009; accepted November 6, 2009)

The subject of this paper is a new software simulating ultrasound signal scattered on moving blood cells during Doppler examination of blood flow velocity using pulsed technique. Generated data are used for optimization and validation of Doppler signals processing algorithms.

The algorithm is based on the finite elements method FEM. A rigorous set of postulates which simplifies physics of modeled phenomenon enables to quicken the program significantly while preserving important properties (from application point of view) of generated signal.

The paper includes description of Doppler RF signal generation algorithm. The simplifying postulates are listed together with resulting signal fidelity degradation. Finally generated raw data is presented together with its Doppler Audio and Color processed version.

The signal processing results enable to reconstruct correctly the velocity profile and its time dependence. The results clearly confirm that the data generated by the algorithm are suitable for Doppler signals processing.

Keywords: RF signal simulation, scattering on blood cells, pulsed Doppler.

1. Introduction

Numerical simulations are a useful tool which enables to get results similar to real data. Its low cost, simplicity and flexibility of the model makes this tool prevalent in many fields. In Doppler ultrasound it is used from the moment when work on a new apparatus begins (examining of processing algorithms, designing probes) till it is introduced to medical society (training simulations).

Despite being relatively short, the history of numerical Doppler signal simulations is rich. MO and COBBOLD [8], VAN LEEUWEN [12] and TALHAMI and KITNEY [11] introduced random nature of Doppler signals in their simulations but not necessarily based on physics of real phenomena. AZIMI and KAK [1] proposed
a point scattering model which was based on the physics but did not take into account random phase of the signal due to the transit time effect. JONES and GIDDENS [4] implemented an algorithm which simulated the signal as a composition of echoes from large number of scattering points. Simulated data included most of the important properties of Doppler signals scattered in blood however it assumed very simplified flow of blood (laminar and steady). A model introduced by WENDLING [13] was an extension of the previous one – pulsatile flow simulations were enabled. Further models by OUNG and FORSBERG [10] assumed much more complicated flows, which were solved using CFD tools (Computational Fluid Dynamics). One of the latest models by KHOSHNIAT [6] enabled to generate Doppler spectrograms efficiently. However, the time required for computation was shorten compromising the simplicity of the physical description of the model [3].

The proposed algorithm is close to the model developed by JONES and GIDDENS’ [4] as far as signal properties are considered. Collected data are used to verify processing algorithms in transcranial Doppler system developed in our laboratory. Simplification of some phenomena of less importance and efforts to make the computational structure as efficient as possible enable to quicken the program significantly while preserving important properties of generated data.

2. Scattering model

The program calculates the signal resulting from scattering of ultrasonic pulse on blood cells moving through the sample volume. By sample volume we mean the volume which is covered by spatial distribution of ultrasound beam. It is possible to implement an algorithm that would consider all known phenomena accompanying propagation of the ultrasound wave in tissue but such an algorithm would be obviously very complicated and using it to generate RF echo data would be very time-consuming. In order to decrease complexity of the problem and to accelerate the execution of the algorithm, following assumptions are made:

• Areas filled with randomly distributed blood cells are represented by uniformly distributed elements (nodes of rectangular grid) with random scattering coefficient. Values of the coefficient are subjected to Rayleigh distribution and once chosen, remain constant for every element. Assumption of this constancy is close to the truth because blood cells transit time through the sample volume is a few milliseconds. We can assume that in such short time blood cells spatial distribution does not change. Elements which represent walls of the vessel have constant value of scattering coefficient, much higher than in the case of blood, which is to simulate a strong echo from the vessel. Size of the finite elements of 20 µm (size of erythrocyte is a few micrometers) and its uniform distribution can negatively influence the signal but the influence is still not precisely described.
• Scattering phenomenon is replaced by partial reflection. In reality, the signal \( S_R \) scattered on a blood cell is a convolution of excitation signal \( S_N \) and an impulse response of the blood cell \( h(t) \).

\[
S_R(t) = S_N(t) * h(t),
\]

(1)

• Without knowledge on the impulse response we have to use a simplified scattering model and to use delta function \( \delta_D(t) \) instead. Amplitude \( A_\delta \) of delta depends on scattering coefficient. Convolution of signal with delta function is an equivalent to multiplication of the signal by delta amplitude \( A_\delta \):

\[
S_R(t) = S_N(t) * \delta_D(t),
\]

(2)

\[
S_R(t) = S_N(t) \cdot A_\delta.
\]

(3)

Hence we introduced the possibility of replacing the scattering phenomenon (described by convolution) by partial reflection (described by multiplication).

• Influence of multiple scattering is neglected (Born approximation). A small part of excitation wave energy is scattered, so energy of signal which was scattered many times is significantly lower. Moreover, multiple scattering signal is a contribution to noise.

• Movement of the scatterers is performed as jumps between successive acquisitions. Movement during interaction of excitation pulse with the elements of the model is neglected – there is no need to consider classic Doppler shift because generated data are for pulsed Doppler examinations only.

• Phenomena of absorption and refraction of ultrasound wave are omitted. Attenuation due to scattering on model’s elements is neglected as well.

• An ultrasound beam is parallel and has a cylindrical shape. A profile of acoustic pressure amplitudes is arbitrary – in this examination a Gaussian profile was used. Assumption of parallelism of the beam results in neglecting of the geometric spectral broadening.

• A blood vessel has a cylindrical shape and a blood flow is axial. A velocity profile and a variance of the velocity in time are arbitrary – in this examination a parabolic profile is used. Real blood flows are much more complicated and often nonuniform.

3. Description of the algorithm

The successive calculation of the algorithm can be divided into three major steps:
1. Sample volume model allocation;
2. Scatterers relocation;
First step is made at the beginning of simulation. After creating sample volume model, steps 2 and 3 occur alternately. In this way the scattered signal from moving scatterers is calculated.

### 3.1. Sample volume allocation

A sample volume is the common space of blood vessel and ultrasound beam (common part of two overlaid cylinders). Model of this volume is built as a 3D matrix (Fig. 1). Matrix elements represent scattering areas (groups of blood cells) and include random values of scattering coefficients. According to the assumptions, scattering phenomenon is replaced by partial reflection so scattering elements will be named reflectors.

![Fig. 1. Shape of modeled volume and its numerical structure. Layers of scattering elements of slopingly orientated sample volume are crept one over another so that they form ellipse based cylinder which is next completed to a cuboid. 2D matrixes store values of parameters which are dependent on pairs of coordinates: \( p_{\text{norm}} \) – acoustic pressure, \( v \) – flow speed, \( t_R \) – delay of backscattered pulse.](image)

Layers of reflectors are crept one on another so that they form an elliptic cylinder oriented along \( y \) axis. 3D matrix shown in Fig. 1 matches to this cylinder after completion to a cuboid. The aim of a described transformation of a sample volume shape is to simplify the algorithm calculation structure.

Apart from 3D matrix with scattering coefficient values, there are also three 2D matrixes which store data dependent on two coordinates only. Matrix in surface:
- X-Y stores values of flow speed \(- v(x, y)\).
- X-Z stores amplitudes of acoustic pressures \(- p_{\text{norm}}(x, z)\).
- Y-Z stores values of delay \(- t_R(y, z)\).

A resolution of the model is a compromise between simplicity of calculations and realistic projection of physical phenomena. For this consideration distance between consecutive reflectors is 20 \( \mu \text{m} \).
3.2. Scatterers relocation

Relocations of reflectors during time $T$ between successive emissions are realized by moving values of scattering coefficients inside sample volume matrix. The distance of replacement along $z$ axis depends on transient speed of blood flow for particular coordinates $(x, y)$ and on the time interval $T$ between consecutive pulses. Some scattering coefficients are lost – reflectors, to whom these coefficients are assigned, leave the sample volume. At the same time new reflectors enter the sample volume. For these elements new values of scattering coefficient are chosen.

3.3. Backscattered signal calculation

Thanks to the assumption of single scattering, superposition can be used to calculate the received signal. The algorithm estimates the signal as a sum of pulses reflected from all elements of the model:

$$S_R(t) = \sum_{x,y,z} S_{RN}(x, y, z, t),$$

where $S_R(t)$ – received signal, $S_{RN}(x, y, z, t)$ – received signal from particular reflector. According to the simplified assumptions (scattering replaced by reflection, parallel beam, no Doppler shift of frequency), the reflected signal is delayed and scaled version of excitation pulse:

$$S_{RN}(x, y, z, t) = S_E(t - t_R(x, y, z)) \cdot p_{\text{norm}}(x, y, z) \cdot \sigma(x, y, z),$$

where $S_E(t)$ – transmitted signal, $t_R(x, y, z)$ – time delay of signal received from particular reflector, $p_{\text{norm}}(x, y, z) = p(x, y, z)/p_{\text{max}}$ – normalized acoustic pressure in particular reflector’s position, $\sigma(x, y, z)$ – relative scattering coefficient of particular reflector. Assumption of a beam parallelism causes that the delay does not depend on $x$ coordinate. Therefore estimation process can be improved further on – calculations along $x$ coordinate can be carried out on vectors instead of scalars, which substantially quicken the algorithm.

$$S_R(t) = \sum_{y,z} S_{RN}(y, z, t),$$

$$S_{RN}(y, z, t) = S_E(t - t_R(y, z)) \cdot \sum_{x} [p_{\text{norm}}(x, y, z) \cdot \sigma(x, y, z)].$$

4. Results of the simulation

Generated data are stored in a 2D matrix where successive rows represent time response of numerical model to successive excitation pulses.
Sloping lines shown in Fig. 2a correlate with moving elements of the model. The more horizontal the lines are (near axis of the vessel), the faster the blood flow is.

Fig. 2. a) Part of algorithm output data matrix, b) response of the model to single pulse excitation (one row from output data matrix).

At the beginning and at the end of response of the model there is significant offset. This occurs when excitation pulse enters or leaves modeled volume. When excitation pulse moves from non-scattering medium to medium with positive mean scattering coefficient value (which takes place in simulation), only that part of the pulse which is in the scattering medium is taken into calculation of backscattered signal. Assuming that scattering coefficient distribution in the medium is close to uniform, the most important for offset to occur is relation between areas which scatter (at particular moment) positive and negative half-periods of the sine wave of the excitation pulse. E.g. as the pulse enters blood vessel and exactly one period of sine wave interacts with blood, acquired signal should be close to zero – backscattered signals from positive and negative half-periods cancel each other. After some time next half-period of the pulse enters blood vessel and there are two positive and one negative half-periods which results in positive value of backscattered signal’s amplitude. As following half-periods enter blood vessel, the amplitude of backscattered signal changes between zero and some positive value which is equal to occurrence of offset. As soon as whole excitation pulse enters scattering medium, offset disappears (on condition that the excitation pulse has no offset).

In order to verify properties of generated signal the data were processed using Doppler – Audio and Doppler – Color algorithms. The data were calculated for the parabolic profiled blood flow. Maximal flow speed was assumed to change between 0.2 and 0.7 m/s as shown in Fig. 3b. The center frequency of the probing pulses was equal to 2 MHz, the speed of sound is 1540 m/s, PRF = 4 kHz. The angle between ultrasound beam and vessel's axis was 60°. For these values a maximum flow speed without aliasing is 1.54 m/s.
Fig. 3. a) Generated data processed with Doppler – Audio algorithm, b) initial maximum flow velocity.

Processing of the signal with a Doppler – Audio algorithm gives results shown in Fig. 3a. Spectrogram is calculated for audio gate placed on the axis of the vessel.

Spectrogram (Fig. 3a) clearly correlates with flow speed function (Fig. 3b) used in the simulation. Substantial blur (spectral broadening) of Doppler frequency is caused by the finite size of excitation pulse, which implies transit time effect. Simultaneous pulse interaction with reflectors moving at different speed also increase the spectral broadening.

Fig. 4. a) Generated data processed with Doppler – Color algorithm, b) time-space distribution of the simulated flow velocity.

Applying a Doppler – Color algorithm to the signal gives the results shown in Fig. 4a. Processed data clearly correlate with the original function of flow speed (Fig. 4b). This means that blood flow information is well contained in generated signal. Noise which is clearly visible in Fig. 4a is a result of random nature of Doppler signals scattered on blood and it is present in real examinations.
5. Conclusions

Considering simplicity of the algorithm, results of the simulations are surprisingly good. Doppler signal and its random nature are well included in the generated data which have a qualitative character.

The proposed algorithm can be easily improved but even its present form properly reflects the space-time simulation of the Doppler recording of the flow signal and will be used in future development of the Doppler devices.

Acknowledgment

This article is an extended version of the paper presented at the 56th Open Seminar on Acoustics – OSA2009, September 15–18 in Goniądz.

References