Noise Elimination of a Multi-tone Broadband Noise with Hybrid Helmholtz Mufflers Using a Simulated Annealing Method

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Noise control is essential in an enclosed machine room where the noise level has to comply with the occupational safety and health act. In order to overcome a pure tone noise with a high peak value that is harmful to human hearing, a traditional reactive muffler has been used. However, the traditional method for designing a reactive muffler has proven to be time-consuming and insufficient. In order to efficiently reduce the peak noise level, interest in shape optimization of a Helmholtz muffler is coming to the forefront.

Helmholtz mufflers that deal with a pure tone have been adequately researched. However, the shape optimization of multi-chamber Helmholtz mufflers that deal with a broadband noise hybridized with multiple tones within a constrained space has been mostly ignored. Therefore, this study analyzes the sound transmission loss (STL) and the best optimized design for a hybrid Helmholtz muffler under a space-constrained situation. On the basis of the plane wave theory, the four-pole system matrix used to evaluate the acoustic performance of a multi-tone hybrid Helmholtz muffler is presented. Two numerical cases for eliminating one/two tone noises emitted from a machine room using six kinds of mufflers (muffler A∼F) is also introduced. To find the best acoustical performance of a space-constrained muffler, a numerical assessment using a simulated annealing (SA) method is adopted. Before the SA operation can be carried out, the accuracy of the mathematical model has been checked using the experimental data. Eliminating a broadband noise hybridized with a pure tone (130 Hz) in Case I reveals that muffler C composed of a one-chamber Helmholtz Resonator and a one-chamber dissipative element has a noise reduction of 54.9 (dB). Moreover, as indicated in Case II, muffler F, a two-chamber Helmholtz Resonator and a one-chamber dissipative element, has a noise reduction of 69.7 (dB). Obviously, the peak values of the pure tones in Case I and Case II are efficiently reduced after the muffler is added.

Consequently, a successful approach in eliminating a broadband noise hybridized with multiple tones using optimally shaped hybrid Helmholtz mufflers and a simulated annealing method within a constrained space is demonstrated.

Keywords: multiple tones, hybrid, Helmholtz, four-pole transfer matrix method, SA method.

Notations

c_o – sound speed (m s^{-1}),
C_{res} – the acoustic compliance \((= \frac{V_c}{\rho_o c_o^2})\),
DD, DD_1 – diameter of the inner tubes (m),
D_\alpha – diameter of the outer chamber (m),
f – cyclic frequency (Hz),
d_H – the diameter of a perforated hole on the perforated tube (m),
HR – Helmholtz Resonator,
it – maximum iteration in SA,
k_o – wave number \((= \frac{\omega}{c_o})\),
kk – cooling rate in SA,
L_k – the length of the resonating neck (m),
L_{res} – the acoustic inertia \((= \frac{\rho_o (L_k + \ell_T)}{S_k})\),
\ell_T – the total end correction \((= \ell_1 + \ell_2 = 1.698 r_k)\) (m),
OBJ – objective function (dB),
p – acoustic pressure (Pa),
p_p – the porosity of the perforated tube in the dissipative element,
Q – volume flow rate of venting gas (m^3 s^{-1}),
r_k – the radius of the resonating tube in the Helmholtz resonator (m),
R_{res} – the acoustic resistance,
SA – simulated annealing,
S_i – the section area at the i-th node (m^2),
S_d – the section area of the main duct connected to the Helmholtz resonator (m^2),

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As pointed out by the Occupational Safety and Health Act (OSHA) of 1970, high pure tone noise levels can be harmful to workers and can not only lead to psychological but also to physiological ailments. Consequently, noise control work on equipment becomes essential. However, in practical engineering work, the constrained problem is mostly concerned with the necessity of operation and maintenance where there is a growing need to optimize the acoustical performance within a confined space. Moreover, a broadband noise hybridized with multiple tones that will lead to psychological and physiological ailments becomes essential. However, in the above research, the assessment in finding a muffler’s optimal shape design for a broadband noise hybridized with multiple tones within a constrained space has been somewhat neglected. In order to promote the best acoustical performance in mufflers, three kinds of hybrid two-chamber mufflers (muffler A: a one-chamber HR element and a one-chamber simple expansion element; muffler B: a one-chamber HR element and a one-chamber tube-extended element; muffler C: a one-chamber HR element and a one-chamber dissipative element) used in eliminating a broadband noise hybridized with one tone will be proposed. Also, in order to reduce a broadband noise hybridized with two tones, three kinds of hybrid three-chamber mufflers (muffler D: a two-chamber HR element and a one-chamber simple expansion element; muffler E: a two-chamber HR element and a one-chamber tube-extended element; muffler F: a two-chamber HR element and a one-chamber dissipative element) within a fixed space are also presented. To facilitate the numerical assessment, two different SA techniques (a cooling rate and an iteration) are adopted. By adjusting the muffler’s shape, varying the acoustical element, and using the SA, the optimal acoustical performance of the mufflers can be achieved.

2. Theoretical background

As illustrated in Fig. 1, there is a broadband noise hybridized with multiple tones in the constrained ma-
The machine room. The three kinds of mufflers A–C shown in Fig. 2 were adopted in dealing with a broadband noise hybridized with one tone (130 Hz). Moreover, three kinds of mufflers (muffler D–F) shown in Fig. 3 were adopted in dealing with a broadband noise hybridized with two tones (130 Hz and 235 Hz). The detailed mathematical derivation of various muffler systems is presented below.

2.1. Muffler A (a one-chamber HR element and a one-chamber simple expansion element)

As derived in previous work (CHIU, CHANG, 2008a, 2008b; CHIU, 2008; 2010a; 2010b; 2010c) and Appendix A, individual transfer matrixes with respect to straight ducts, an HR element, a sudden expanded duct, a sudden contracted duct, and a dissipative duct are described as follows:

\[
\begin{align*}
\left( \frac{p_1}{\rho_o c_o u_1} \right) &= f_1(L_1, L_3) \left[ T_{S1, 1_{1,1}} T_{S1, 1_{1,2}} \left( \frac{p_2}{\rho_o c_o u_2} \right) \right], \\
\left( \frac{p_2}{\rho_o c_o u_2} \right) &= \left[ \text{THR}, 1_{2,1} \text{THR}, 2_{1,2} \left( \frac{p_3}{\rho_o c_o u_3} \right) \right], \\
\left( \frac{p_3}{\rho_o c_o u_3} \right) &= f_2(L_3) \left[ T_{S1, 3_{1,1}} T_{S1, 3_{1,2}} \left( \frac{p_4}{\rho_o c_o u_4} \right) \right], \\
\left( \frac{p_4}{\rho_o c_o u_4} \right) &= \left[ \text{TSE}, 1_{4,1,1} \text{TSE}, 1_{4,2,2} \left( \frac{p_5}{\rho_o c_o u_5} \right) \right], \\
\left( \frac{p_5}{\rho_o c_o u_5} \right) &= f_3(L_4) \left[ T_{S1, 5_{1,1}} T_{S1, 5_{1,2}} \left( \frac{p_6}{\rho_o c_o u_6} \right) \right], \\
\left( \frac{p_6}{\rho_o c_o u_6} \right) &= \left[ \text{TSC}, 1_{5,1,1} \text{TSC}, 1_{5,2,2} \left( \frac{p_7}{\rho_o c_o u_7} \right) \right], \\
\left( \frac{p_7}{\rho_o c_o u_7} \right) &= f_4(L_2) \left[ T_{S1, 7_{1,1}} T_{S1, 7_{1,2}} \left( \frac{p_8}{\rho_o c_o u_8} \right) \right].
\end{align*}
\]

The total transfer matrix assembled by multiplication is simplified as

\[
\left\{ \frac{p_1}{\rho_o c_o u_1} \right\} = \prod_m [T_m(f)] \left\{ \frac{p_8}{\rho_o c_o u_8} \right\}.
\]

The sound transmission loss (STL) of a muffler is defined as (MUNJAL, 1987)

\[
\text{STL}_1(Q, f, RT_1^A, RT_2^A, RT_3^A, RT_4^A) = 20 \log \left( \frac{1}{2} T_{11} + T_{12} + T_{21} + T_{22} \right) + 10 \log \left( \frac{S_p}{S_b} \right).
\]
2.2. Muffler B (a one-chamber HR element and a one-chamber tube-extended element) (CHU, 2010a; 2010b; 2010c)

Similarly, the sound transmission loss (STL) is

\[ STL_2(Q, f, RT_1^B, RT_2^B, RT_3^B, RT_4^B, RT_5^B, RT_6^B) = 20 \log \left( \frac{1}{2} (T_{11} + T_{12} + T_{21} + T_{22}) \right) + 10 \log \left( \frac{S_1}{S_{10}} \right). \] (10)

2.3. Muffler C (a one-chamber HR element and a one-chamber dissipative element) (CHU, 2011a; 2011b; 2012)

Also, the sound transmission loss (STL) is

\[ STL_3(Q, f, RT_1^C, RT_2^C, RT_3^C, RT_4^C, RT_5^C, RT_6^C) = 20 \log \left( \frac{1}{2} (T_{11} + T_{12} + T_{21} + T_{22}) \right) + 10 \log \left( \frac{S_1}{S_6} \right). \] (11)

2.4. Muffler D (a two-chamber HR element and a one-chamber simple expansion element)

Likewise, the related sound transmission loss (STL) is

\[ STL_4(Q, f, RT_1^D, RT_2^D, RT_3^D, RT_4^D, RT_5^D, RT_6^D) = 20 \log \left( \frac{1}{2} (T_{11} + T_{12} + T_{21} + T_{22}) \right) + 10 \log \left( \frac{S_1}{S_{10}} \right). \] (12)

2.5. Muffler E (a two-chamber HR element and a one-chamber tube-extended element)

Equally, the sound transmission loss (STL) is

\[ STL_5(Q, f, RT_1^E, RT_2^E, RT_3^E, RT_4^E, RT_5^E, RT_6^E, RT_7^E, RT_8^E) = 20 \log \left( \frac{1}{2} (T_{11} + T_{12} + T_{21} + T_{22}) \right) + 10 \log \left( \frac{S_1}{S_{12}} \right). \] (13)

2.6. Muffler F (a two-chamber HR element and a one-chamber dissipative element)

Also, the sound transmission loss (STL) is

\[ STL_6(Q, f, RT_1^F, RT_2^F, RT_3^F, RT_4^F, RT_5^F, RT_6^F, RT_7^F) = 20 \log \left( \frac{1}{2} (T_{11} + T_{12} + T_{21} + T_{22}) \right) + 10 \log \left( \frac{S_1}{S_8} \right). \] (14)

2.7. Overall sound power level

The overall SWL\(_T\) silenced by the muffler at the outlet is

\[ SWL_T = 10 \log_{10} \left( \sum_m 10^{(SWL(f_m) - STL(f_m)) / 10} \right), \] (15)

where \(SWL(f_m)\) is the original SWL at the inlet of the muffler (or pipe outlet), and \(m\) is the index of the octave band frequency. \(STL(f_m)\) is the muffler’s STL with respect to the relative octave band frequency.

2.8. Objective function

By using the formulas of Eqs. (9)–(15), the objective function used in the S4 optimization with respect to each type of muffler was established. For muffler A, the objective function in minimizing overall SWL\(_T\) hybridized with one tone \((f_1)\) is

\[ OBJ_1 = SWL_1(Q, f_1, RT_1^A, RT_2^A, RT_3^A, RT_4^A). \] (16)

For muffler B, the objective function in minimizing overall SWL\(_T\) hybridized with one tone \((f_1)\) is

\[ OBJ_2 = SWL_2(Q, f_1, RT_1^B, RT_2^B, RT_3^B, RT_4^B, RT_5^B, RT_6^B). \] (17)

For muffler C, the objective function in minimizing overall SWL\(_T\) hybridized with one tone \((f_1)\) is

\[ OBJ_3 = SWL_3(Q, f, RT_1^C, RT_2^C, RT_3^C, RT_4^C, RT_5^C). \] (18)

For muffler D, the objective function in minimizing overall SWL\(_T\) hybridized with two tones \((f_1 \text{ and } f_2)\) is

\[ OBJ_4 = SWL_4(Q, f_1, f_2, RT_1^D, RT_2^D, RT_3^D, RT_4^D, RT_5^D, RT_6^D). \] (19)

For muffler E, the objective function in minimizing overall SWL\(_T\) hybridized with two tones \((f_1 \text{ and } f_2)\) is
OBJ_5 = SWL_5(Q, f_1, f_2, RT^{E}_{1}, RT^{E}_{2}, RT^{E}_{3}, RT^{E}_{4},
RT^{E}_{5}, RT^{E}_{6}, RT^{E}_{7}, RT^{E}_{8}).

(20)

For muffler F, the objective function in minimizing overall SWL hybridized with two tones (f_1 and f_2) is

OBJ_6 = SWL_6(Q, f_1, f_2, RT^{E}_{1}, RT^{E}_{2}, RT^{E}_{3}, RT^{E}_{4}, RT^{E}_{5}, RT^{E}_{6}, RT^{E}_{7}).

(21)

3. Model check

Before performing the SA optimal simulation on mufflers, an accuracy check of the mathematical model on the HR element is performed using the experimental data from Selamet et al. (2005). As depicted in Fig. 4, the performance curve with respect to the theoretical and experimental data is relatively accurate and in agreement. Therefore, the proposed fundamental mathematical model with the HR element is acceptable. Consequently, the model linked with the numerical method is applied to the shape optimization in the following section.

4. Case studies

In this paper, a machine room within a constrained space is shown in Fig. 1. As shown in Fig. 1, the available space for a muffler is 0.5 m in width, 0.5 m in height, and 1.2 m in length. In Case I, the broadband noise is hybridized with one tone (130 Hz). In dealing with the broadband mixed with a one-tone noise, three kinds of two-chamber mufflers (muffler A: a one-chamber HR element and a one-chamber simple expansion element; muffler B: a one-chamber HR element and a one-chamber tube-extended element; muffler C: a one-chamber HR element and a one-chamber dissipative element) shown in Fig. 2 are proposed. Moreover, in Case II, the broadband noise is hybridized with two tones (130 Hz and 235 Hz). In dealing with the broadband mixed with a two-tone noise, three kinds of three-chamber mufflers (muffler D: a two-chamber HR element and a one-chamber simple expansion element; muffler E: a two-chamber HR element and a one-chamber tube-extended element; muffler F: a two-chamber HR element and a one-chamber dissipative element) shown in Fig. 3 are proposed. The related sound power level inside the muffler inlet for Case I and Case II are shown in Table 1. The ranges of the parameters of the mufflers are listed in Table 2. The flowing resistance (σ_{fr}) of the absorbing material filled in the dissipative element is assumed to be 22000 ralys/m. In the existing venting system, the flow rate (Q) is given as 0.01 (m³/s).

Table 1. The original Sound Power Level in Case I and Case II.

<table>
<thead>
<tr>
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<tbody>
<tr>
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</tr>
<tr>
<td>2000</td>
<td>104</td>
<td>2000</td>
<td>104</td>
</tr>
<tr>
<td>Overall</td>
<td>139.4</td>
<td>Overall</td>
<td>141.1</td>
</tr>
</tbody>
</table>

Fig. 4. Performance of a single-chamber Helmholtz muffler without the mean flow [experimental data is from Selamet et al. (2005)].
Table 2. The ranges of parameters for mufflers A–F.

<table>
<thead>
<tr>
<th>Item</th>
<th>ranges of parameters</th>
</tr>
</thead>
<tbody>
<tr>
<td>Muffler A</td>
<td>(RT_1^4 = [0.2, 0.4]; RT_2^4 = [0.003, 0.02]; RT_3^4 = [0.02, 0.05]; RT_4^4 = [0.2, 0.4])</td>
</tr>
<tr>
<td>Muffler B</td>
<td>(RT_1^B = [0.2, 0.4]; RT_2^B = [0.003, 0.02]; RT_3^B = [0.02, 0.05]; RT_4^B = [0.2, 0.4])</td>
</tr>
<tr>
<td>Muffler C</td>
<td>(RT_1^C = [0.2, 0.4]; RT_2^C = [0.003, 0.02]; RT_3^C = [0.02, 0.05]; RT_4^C = [0.00175, 0.007]; RT_5^C = [0.03, 0.1])</td>
</tr>
<tr>
<td>Muffler D</td>
<td>(RT_1^D = [0.2, 0.4]; RT_2^D = [0.003, 0.02]; RT_3^D = [0.02, 0.05]; RT_4^D = [0.2, 0.4])</td>
</tr>
<tr>
<td>Muffler E</td>
<td>(RT_1^E = [0.2, 0.4]; RT_2^E = [0.003, 0.02]; RT_3^E = [0.02, 0.05]; RT_4^E = [0.2, 0.4]; RT_5^E = [0.2, 0.4])</td>
</tr>
<tr>
<td>Muffler F</td>
<td>(RT_1^F = [0.2, 0.4]; RT_2^F = [0.003, 0.02]; RT_3^F = [0.02, 0.05]; RT_4^F = [0.00175, 0.007]; RT_5^F = [0.03, 0.1])</td>
</tr>
</tbody>
</table>

5. Simulated annealing

The fundamental concept of simulated annealing (SA), a local search process which imitates the softness process (annealing) of metal and is started by generating a random initial solution, was first introduced by Metropolis et al. (1953) and subsequently developed by Kirkpatrick et al. (1983). The SA scheme is a variation of the hill-climbing algorithm where all downhill movements are accepted for the decrement of the system’s energy. However, in order to escape the local optimum, solutions that are inferior (uphill moves) to the current one are allowed. Here, the optimal process is dominated by two primary parameters – \(kk\) and iter; for next steps of optimizing calculation of the cases considered, the SA procedure described in the previous paper has been applied (Chiu, 2008a; 2009a; 2010a; 2010b; 2011a; Chiu, Chang, 2008b) where \(kk\) is the cooling rate and iter is the maximal iteration. The SA process is repeated until the predetermined number (iter) of the outer loop is reached.

6. Results and discussion

6.1. Results

The accuracy of the SA optimization depends on the cooling rate (\(kk\)) and the number of iterations (iter). To achieve a good optimization, the range of both the cooling rate and the number of iterations are set as \(kk = [0.91, 0.93, 0.95, 0.97, 0.99]\) and iter = [50, 100, 500, 1000].

An optimization for muffler B in reducing the broadband noise in Case I by varying the \(kk\) and the iter has been approached and shown in Table 3. Using Table 3, the optimal STL curves with respect to various SA parameters (\(kk\) and iter) are plotted and depicted in Figs. 5–6. As indicated in Table 3 and Figs. 5–6, the best result occurs at the eighth set when \(kk = 0.99\) and iter = 1000 are applied. Applying the above best SA parameter set in the optimization of muffler A and muffler C, the resultant design parameters and minimized SWLs have been obtained and shown in Table 4. Using the optimal design data of mufflers A–C in Table 4, the optimal STL curves with respect to the original SWL curve are plotted and depicted in Fig. 7.

Table 3. The optimal design data with respect to various SA parameters (\(kk\), iter) for muffler B.

<table>
<thead>
<tr>
<th>Item</th>
<th>SA parameters</th>
<th>Results</th>
</tr>
</thead>
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<tr>
<td></td>
<td>(kk)</td>
<td>iter</td>
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<tr>
<td>1</td>
<td>0.91</td>
<td>50</td>
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<tr>
<td>2</td>
<td>0.93</td>
<td>50</td>
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<tr>
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<td>0.95</td>
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</tr>
<tr>
<td>4</td>
<td>0.97</td>
<td>50</td>
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<tr>
<td>5</td>
<td>0.99</td>
<td>50</td>
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<tr>
<td>6</td>
<td>1.00</td>
<td>50</td>
</tr>
<tr>
<td>7</td>
<td>0.99</td>
<td>500</td>
</tr>
<tr>
<td>8</td>
<td>0.99</td>
<td>1000</td>
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</table>
Table 4. Optimal design data with respect to various mufflers (kk = 0.99; iter = 1000).

<table>
<thead>
<tr>
<th>Muffler</th>
<th>$RT_1^A$</th>
<th>$RT_2^A$</th>
<th>$RT_3^A$</th>
<th>$RT_4^A$</th>
<th>$RT_5^A$</th>
<th>$RT_6^A$</th>
<th>SWL [dB]</th>
</tr>
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<tbody>
<tr>
<td>Muffler A</td>
<td>0.2251</td>
<td>0.00513</td>
<td>0.02376</td>
<td>0.2251</td>
<td>0.2251</td>
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<td>92.9</td>
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<tr>
<td>Muffler B</td>
<td>$RT_1^B$</td>
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<td>$RT_6^B$</td>
<td>SWL2 [dB]</td>
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<td>Muffler C</td>
<td>$RT_1^C$</td>
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<td>$RT_3^C$</td>
<td>$RT_4^C$</td>
<td>$RT_5^C$</td>
<td>$RT_6^C$</td>
<td>SWL3 [dB]</td>
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<tr>
<td>Muffler D</td>
<td>$RT_1^D$</td>
<td>$RT_2^D$</td>
<td>$RT_3^D$</td>
<td>$RT_4^D$</td>
<td>$RT_5^D$</td>
<td>$RT_6^D$</td>
<td>SWL4 [dB]</td>
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<tr>
<td>Muffler E</td>
<td>$RT_1^E$</td>
<td>$RT_2^E$</td>
<td>$RT_3^E$</td>
<td>$RT_4^E$</td>
<td>$RT_5^E$</td>
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<td>$RT_5^F$</td>
<td>$RT_6^F$</td>
<td>SWL6 [dB]</td>
</tr>
</tbody>
</table>

Note:
Muffler A – $RT_4^A = DD/D_0$; $RT_3^A = dd_1$; $RT_2^A = Lk_1$; $RT_1^A = DD_1/D_0$.
Muffler B – $RT_4^B = DD/D_0$; $RT_3^B = dd_1$; $RT_2^B = Lk_1$; $RT_1^B = DD_1/D_0$; $RT_2^B = L_5/L_4$; $RT_4^B = L_7/L_4$.
Muffler C – $RT_4^C = DD/D_0$; $RT_3^C = dd_1$; $RT_2^C = Lk_1$; $RT_1^C = dh_5$; $RT_2^C = p\%$.
Muffler D – $RT_4^D = DD/D_0$; $RT_3^D = dd_4$; $RT_2^D = d_2$; $RT_1^D = L_5/K_3$; $RT_2^D = DD_2/D_3$.
Muffler E – $RT_4^E = DD/D_0$; $RT_3^E = dd_1$; $RT_2^E = d_2$; $RT_1^E = L_5/K_3$; $RT_2^E = LK_2$; $RT_4^E = DD_1/D_0$; $RT_5^E = L_6/L_5$; $RT_6^E = L_8/L_5$.
Muffler F – $RT_4^F = DD/D_0$; $RT_3^F = dd_4$; $RT_2^F = d_2$; $RT_1^F = LK_2$; $RT_2^F = LK_2$; $RT_2^F = dd_4$; $RT_1^F = DH_5$; $RT_5^F = p\%$.

6.2. Discussion

Similarly, using the same SA parameter set in Case II, the optimization for mufflers D–F has been approached and shown in Table 4. Using the optimal design data of mufflers D–F in Table 4, the optimal STL curves with respect to the original SWL curve are plotted and depicted in Fig. 8.

As described in Subsec. 6.1, in seeking a better STL with three kinds of mufflers (mufflers A–C) in Case I, a shape optimization process in conjunction...
with the SA optimizer is performed. As indicated in Table 1 and Table 4, the SWL at the venting outlet will be reduced from 139.4 dB to 92.9 when muffler A is added. The SWL at the venting outlet will be reduced from 139.4 dB to 85.0 when muffler B is adopted. Similarly, the SWL at the venting outlet will be reduced from 139.4 dB to 82.5 when muffler C is used. As indicated in Fig. 7, it is obvious that the peak value at a pure tone of 130 Hz has been reduced by using the HR element in mufflers A–C. Moreover, considering the overall acoustical effect, muffler C, having a one-chamber HR equipped with a dissipative element, is superior to muffler A (a one-chamber HR equipped with a one-chamber simple expansion element) and muffler B (a two-chamber HR equipped with a one-chamber tube-extended element).

Similarly, a shape optimization process in Case II is performed. The results shown in Table 4 indicate that the SWL at the venting outlet will be reduced from 141.1 dB to 91.1 when muffler D is added. The SWL at the venting outlet will be reduced from 141.1 dB to 80.4 when muffler E is adopted. Also, the SWL at the venting outlet will be reduced from 141.1 dB to 71.4 when muffler F is used. As indicated in Fig. 8, it is obvious that the peak values at the pure tones of 130 Hz and 235 Hz have been fully eliminated by using the two HR elements in mufflers D–F. Moreover, considering the overall acoustical effect, muffler F with a noise reduction of 69.7 dB is superior to the other mufflers. Consequently, the approach used for the optimal noise elimination in the broadband noise hybridized with multiple tones proposed in this study within a limited space is quite important and easily achieved.

7. Conclusions

It has been shown that hybrid HR mufflers used in dealing with a broadband noise hybridized with multiple tones can be easily and efficiently optimized within a limited space by using a four-pole transfer matrix as well as a SA optimizer. Two kinds of SA parameters (kk, iter) play essential roles in the solution’s accuracy during the SA optimization. As indicated in Figs. 5–6, the tuning ability established by adjusting the design parameters of muffler B is reliable.

As investigated in Case I in Sec. 6 and indicated in Fig. 7, the peak value (at 130 Hz) of the SWL at the venting outlet of the machine room can be eliminated by using the one-chamber HR in mufflers A–C. Considering the overall acoustical effect, the overall silenced SWL with respect to mufflers A–C reached 92.9 dB, 85.0 dB, and 82.5 dB. It is obvious that muffler C with a noise reduction of 54.9 dB is superior to the other mufflers. Moreover, for the numerical approach in Case II, Fig. 8 reveals that the peak values (at 130 Hz and 235 Hz) of the SWL at the venting outlet of the machine room also can be eliminated by using two chambers of the HR equipped in mufflers D–F. Similarly, the overall silenced SWL with respect to mufflers D–F reached 91.1 dB, 80.4 dB, and 71.4 dB. It is obvious that muffler F with a noise reduction of 69.7 dB is superior to the other mufflers. Consequently, the approach used for the optimal noise elimination in the broadband noise hybridized with multiple tones proposed in this study within a limited space is quite important and easily achieved.

Appendix A. Transfer matrix of a Helmholtz Resonator Muffler

As indicated in Fig. 9, the one-array Helmholtz Resonator (HR) muffler has one chamber in a sectional direction. Assuming that the characteristic length of the HR element is smaller than the acoustical wave, the neck of the resonator can be regarded as a lumped mass. The air compression is analogous to a spring during the adiabatic resonating process. Considering the acoustical end correction, the factors (ℓ1 and ℓ2) of the end correction with the flange are (LORD, RAYLEIGH, 1945)

$$\ell_1 = \ell_2 = \frac{8r_k}{3\pi} = 0.849r_k.$$  \hspace{1cm} (22)

For a simplified mathematical model of the HR element, the resonant frequency (fc) is

$$f_{\text{res}} = \frac{c_o}{2\pi} \sqrt{\frac{S_k}{V_L(L_k + \ell_T)}},$$ \hspace{1cm} (23)

where $c_o$ is the sound speed, $S_k$ is the section area of the resonating neck, $L_k$ is the length of the resonating neck, and $V_L$ is the volume of the lumped mass.
Substituting Eq. (28) for Eq. (27) yields
\[
\rho_c \nu_3 = \rho_c \nu_1 + \frac{c_o}{S_d Z_r} p_1.
\]
Rearranging Eq. (29) in matrix form yields
\[
\begin{bmatrix}
    p_3 \\
    \rho_c \nu_3
\end{bmatrix} = \begin{bmatrix}
    1 & 0 \\
    \frac{c_o}{S_d Z_r} & 1
\end{bmatrix} \begin{bmatrix}
    p_1 \\
    \rho_c \nu_1
\end{bmatrix}.
\]

The acoustic resistance \( R_{res} \) is ignored as zero when the resonation occurred. The acoustical impedance \( Z_r \) is simplified as
\[
Z_r = j\omega L_{res} + \frac{1}{j\omega C_{res}}.
\]

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