The Characteristic of Sound Reflections from Curved Reflective Panels

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The paper presents the verification of a solution to the narrow sound frequency range problem of flat reflective panels. The analytical, numerical and experimental studies concerned flat panels, panels with curved edges and also semicircular elements. There were compared the characteristics of sound reflected from the studied elements in order to verify which panel will provide effective sound reflection and also scattering in the required band of higher frequencies, i.e. above the upper limit frequency. Based on the conducted analyses, it was found that among some presented solutions to narrow sound frequency range problem, the array composed of panels with curved edges is the most preferred one. Nevertheless, its reflection characteristic does not meet all of the requirements, therefore, it is necessary to search for another solution of canopy which is effective over a wide frequency range.

Keywords: sound reflection, reflective panels, sound scattering.

1. Introduction

In the face of increasing requirements for spaces with acoustic qualities, the research on some reflective structures seems to be indispensable. In the interiors such as concert halls and auditoria the issue of appropriate transferring the first reflection of a sound wave is important (Schroeder, 1979; Cremer, 1989; Ando, 1985) as well as balancing the acoustic energy distribution in the whole space (Beranek, 1996; Kamisiński et al., 2009). These demands may be fulfilled through the application of reflective panels suspended above the stage (Fig. 1). Properly designed reflective structures should provide the sound reflection in the wide frequency range from about 250 Hz to 4 kHz, flat character of obtained frequency characteristic (±3 dB) and spatially uniform reflected sound propagation (Skålevik, 2006). The most commonly used are flat panels, which due to their shape, size and the configuration of elements evoke the problem of narrow frequency range of reflected sound (Szelaż et al., 2013). One of the solution to this issue might be the usage of panels with convex edges or semicircular ones (Rathsam, Wang, 2010). The curved surfaces improve the dispersion at high frequencies and consequently smooth the frequency response in the useful passband. In the paper there was presented the analysis of these approaches.

Fig. 1. The reflective panels which provide appropriate transferring of sound waves’ reflection, Feliks Nowowiejski Warmia and Mazury Philharmonic, Olsztyn (photo by Piotr Pękala).
2. Theory

There are several kinds of models describing the phenomenon of reflection. A large number of them is based on the inhomogeneous Helmholtz-Kirchhoff equation:

\[ \Delta p(r) + k^2 p(r) = -q(r), \]  

where \( p(r) \) is the acoustic pressure dependent only on the spatial variable, \( q(r) \) is the function characterizing the sound source dependent only on the spatial variable, \( k \) is the wave number and \( \Delta \) is the Laplace operator. Naturally, working with the most general models is usually difficult. That is why, there appeared some simplifications and attempts to approximate the formulas. The most commonly used are the Fresnel-Kirchhoff approximation and above all its simplification proposed by Rindel (1986, 1990).

The main assumption connected with these models is that the distances from the reflective panel to the sound source \( (r_0) \) and to the receiver \( (r) \) are large in comparison with the wavelength and the size of the panel. Using the Fresnel-Kirchhoff approximation the reflection from each point of the reflective surface is considered. It means that such an approach is very thorough but at the same time complicated in application. That is why, there were created some further simplifications like the one proposed by Rindel. In this model the focus on the two particular points is assumed: the center and the edge of the panel. The whole analysis which is much simplified leads to an observation that the value of attenuation of sound level after reflection depends mainly on a relative density of the reflective array \( \mu \) and takes the following formula:

\[ \Delta L_{\text{diff}} \cong 20 \log \mu. \]  

Moreover, as a result of mentioned approximations there are obtained the values of two limiting frequencies: the lower \( f_C \) and the upper one \( f_G \), which define the range of effective sound reflections and may be calculated respectively according to the equations:

\[ f_C = \frac{ct}{2S_{\text{panel}} \cos \theta}, \]  
\[ f_G = \frac{ct}{2S_{\text{array}} \cos \theta}, \]

where \( c \) is the speed of sound in the air, \( S_{\text{panel}} \) is the area of the panel, \( \theta \) is the angle of acoustic wave’s incidence, \( S_{\text{array}} \) is the area of the array and \( \tilde{r} \) is a characteristic distance:

\[ \tilde{r} = \frac{2r_0r}{r_0 + r}. \]

The Fresnel-Kirchhoff approximation is not the only one. Due to revaluation of the reflection rate and some inaccuracies while analysing the phenomena at the edge, the approach proposed by Skålevik (2007) was considered. The aim is to identify the border when the scattered acoustic pressure is close to the pressure taken from Fresnel-Kirchhoff theory. The result of the analysis is a low limiting frequency \( f_c \) defined for a circle shape (Bethe, 1944; Rayleigh, 1897) which may be generalized for other shapes owing to the edge density of a panel \( \varepsilon_p \) defined by Skålevik.

All in all, the frequency response of flat reflective structures described as a bandpass filter has two independent low limit frequencies \( f_c \) and \( f_G \) and also the upper limiting frequency \( f_G \). The first low frequency relates to sound waves of a length substantially greater than the dimensions of a single reflective element. It has been specifically derived from the theory of scattering by Skålevik. The second, determined by Rindel, results from the diffraction at the edge of the panel. Finally, the frequency response might be described as two series set filters, named respectively reflective filter and Fresnel-Kirchhoff filter. The first describes the ability of the panel to reflect the acoustic wave, the second determines how much sound reflected from the panels array and reached to the receiver (so-called sensitivity of reflection). Furthermore, basing on the Fresnel-Kirchhoff theory it is possible to determine the attenuation of sound level in the passband. In the case of the ideal reflective filter the sound level in passband is not reduced. However, below the cut-off frequency \( f_c \) the sound level decreases 6 dB per octave. It is a direct consequence of the insufficient size of the panel in comparison with the wavelength. A practical solution would therefore be a combination of the Fresnel-Kirchhoff filter with the high-pass reflective filter (Fig. 2).

Up to this point there were discussed only flat reflective structures. In the case of such elements the incident sound wave is scattered due to the diffraction resulting from the finite-sized panels. Consequently, the reflected sound level decreases. However, it is known that the sound might be also scattered if elements are curved. The curvature induces diffusion of the reflected energy when the surface is convex or focusing when it is concave. If the wavelength is assumed to be small compared to surface size the attenuation associated with the curvature could be accounted for by a simple beam tracing method. To illustrate this case, a rigid cylinder having a radius \( R_c \) is considered (Fig. 3). The reduction in sound level is proportional to the ratio of incident to reflected beam areas (Rindel, 1986). The width of the reflected beam tube at the receiver is \( (r_0 + r) d\beta \). At the image receiver point the beam width might be equal to \( (r_0 + r) d\beta_1 \) if the surface curvature is not included.

Finally, the attenuation due to the surface curvature is:

\[ \Delta L_{\text{curv}} = -10 \log \left( \frac{(r_0^2 + r) d\beta}{(r_0 + r) d\beta_1} \right) = -10 \log \left( \frac{(r_0^2 + r)(d\beta / d\beta_1)}{(r_0 + r)} \right). \]
Moreover, using Fig. 3 one may write that \( r'_0 \, d\beta = r_0 \, d\beta_1 = R_c \, d\varphi \cos \theta \) and \( d\beta = d\beta_1 + 2 \, d\varphi \), and consequently:

\[
\frac{d\beta}{d\beta_1} = 1 + \frac{2r_0}{R_c \cos \theta} \tag{7}
\]

Accordingly, for plane waves the loss of reflected sound intensity might be obtained from the equation:

\[
\Delta L_{\text{curv}} = -10 \log \left| 1 + \frac{\hat{r}}{R_c \cos \theta} \right| \tag{8}
\]

where \( \Delta L \) is the difference between reflected and direct sound levels, \( \Delta L_{\text{dif}} \) is the attenuation of sound level due to diffraction and \( \Delta L_{\text{curv}} \) is the attenuation of sound level due to the surface curvature.

### 3. Methodology

The paper aims at defining the influence of the curvature of a reflective structure on the characteristic of reflected sound. For this purpose experimental research and numerical calculations were carried out.

The experimental study was held in an anechoic chamber where a specially designed measurement setup was installed (Fig. 4). The sound source was located 4 m above a sample and the position of a microphone changed from an angle 0 to 90 degrees (Felis et al., 2012; Batko et al., 2008). The tested samples were rectangular in plan and had two or four edges curved with varied radii (Figs. 5 and 6). The basic
assumption for the research was that the value of a total reflecting area was constant for all compared elements in order to ensure the equal level of reflected energy. Therefore, the reference flat panel as well as the total reflecting area of the samples had dimensions $120 \times 80$ cm. An exception was the sample with four curved edges, its whole surface had a size of $90 \times 90$ cm.

The relative level of sound reflection $L_x$ from the reflective structure was determined as a function of the incident acoustic wave frequency on the basis of the following formula (Kamisiński et al., 2010; 2012a):

$$L_x = 20 \log \left( \frac{F(h(t)_{\text{structure}} - h(t)_{\text{empty}})}{F(h(t)_{\text{ref}} - h(t)_{\text{empty}})} \right),$$

(10)

where $F$ is the Fourier transform, $h(t)_{\text{structure}}$ is the impulse response of a reflective structure, $h(t)_{\text{ref}}$ is the impulse response of reference (flat panel) and $h(t)_{\text{empty}}$ is the impulse response of a measurement setup without tested structures.

Apart from frequency responses, there were also determined the directionalities of sound reflections and diffusion coefficients $d_0$ for the angle of sound wave’s incidence equal to 0 degree. Both characteristics were defined and normalized analogously to frequency responses (i.e. relative to the flat panel), however, a particular diffusion coefficient was calculated based on the autocorrelation function of directionality characteristic.
of the sound wave reflected from the structure (COX, D’ANTONIO, 2004; KAMISIŃSKI et al., 2012b):

\[ d_0 = \frac{\left( \sum_{i=1}^{n} 10^{L_i/10} \right)^2 - \sum_{i=1}^{n} \left( 10^{L_i/10} \right)^2}{\left( n - 1 \right) \sum_{i=1}^{n} \left( 10^{L_i/10} \right)^2} \]

where \( n \) is the number of measurements and \( L_i \) is the sound pressure level in the \( i \)-th point.

The experiment was also supported by numerical calculations which were conducted in two ways: using the Fresnel-Kirchhoff approximation extended by beam tracing method for curved panels as well as applying finite elements method. Both models were carried out as two-dimensional ones which means that reflection characteristics were verified in accordance with a longitudinal section. The first one concerns mathematical calculations based on Fraunhofer solution of Helmholtz-Kirchhoff approximation. Assuming that the surface admittance variation is only in the main directions of propagation of the reflected sound wave, the scattered pressure for the normal incidence on the front surface, \( S \), of the surface admittance variation is only in the

\[ \theta \] is the angle of reflection and \( 2a \times 2b \) are the dimensions of a reflective structure. For the purpose of carrying out this analysis, there was created an implementation of above mathematical formula in the MATLAB environment. The second model based on the finite elements method was prepared using software ABAQUS (SZELĄG, 2014). The studied structures were meshed using a minimum density of six mesh elements per wavelength to ensure a minimal variation of sound pressure across a single element.

4. Experiments and results

In this section, there are discussed some differently shaped reflective panels (flat, with curved edges, semicircular) in the context of obtained frequency characteristic of sound wave reflected from the studied elements. It is known that for the acoustic wave of a frequency exceeding the upper limit \( f_g \), its frequency response becomes highly uneven, since the reflection level depends on the geometrical point of sound wave’s incidence. The fluctuations above a reflective panel’s geometric limit frequency might be avoided if the partial dispersion of sound reflected from appropriately shaped panel’s edges occurs. This solution also reduces the amount of acoustic energy that reaches from one particular direction, which would introduce some coloration into the reflected signal.

The first point of the analysis is to compare the results obtained from measurements, numerical calculations (Finite Element Method) and analytical calculations (Fraunhofer solution). Figure 7 presents the exemplary characteristics of the panel with curved edges of radius 20 cm. Exceptionally in this case, the characteristics were not normalized to a flat panel, but to the maximum level of reflection obtained for this curved panel. As shown in graphs, all obtained characteristics clearly illustrate the basic properties of sound reflection from the sample, i.e. the level of reflection and main directions of propagation of the reflected sound wave. Obviously there are some noticeable differences in the characteristics. For example, the results which based on the Fraunhofer solution differ from the other two in the case of reflection angles close to 90 degrees. On the other hand, the finite element method slightly underestimates reflections in the specular reflection area for low and medium frequencies. Nevertheless, for the purpose of the performed analysis it can be stated that all methods reliably predict the response of reflective structures for the frequency and angle range of interest. However, one should keep in mind all limitations of each method.

The next step of the study was to compare the characteristics of sound reflection from the flat, curved and semicircular panels shown in Fig. 5. The directional characteristics of sound reflection from examined panels for selected frequencies are presented in
Fig. 7. Exemplary directional characteristics of sound reflection from the panel with curved edges of radius 20 cm shown in Fig. 5.

Figs. 8 and 9 for the case of laboratory measurements and calculations using finite element method, respectively. Moreover, Fig. 10 shows the amplitude – frequency characteristics of sound reflection from the studied panels for some selected reflection angles (measured values).

First of all, comparing measured and calculated directional characteristics (Figs. 8 and 9), it is confirmed once again that the finite element method underestimates reflections in the specular reflection area. However, both methods give results that illustrate the characteristic properties of the tested structures’ frequency responses.

Analysing the all obtained graphs one may notice that with increasing frequency of incident wave the reflection from the flat panel becomes highly specular. On the other hand, the more curved panel’s edges are, the more diffuse sound reflection is. As a consequence, the semicircular panel is characterized by a similar reflection effectiveness in each direction. It is also confirmed by appropriate diffusion coefficients shown in Fig. 11. Nevertheless, a major disadvantage of semicir-
The particular element is the low level of the reflected sound. The compromise solution seems to be the panel with a flat surface in the middle and curved edges. Then, the level of specularly reflected sound slightly decreases while the non-specular reflection improves relative to the reflection from a flat panel.

The reflective panel with curved edges was subjected to further verification. This time the study was performed on a sample with four curved edges as shown in Fig. 6. On the basis of laboratory measurements, there was obtained the amplitude-frequency characteristic of the sound reflected specularly for incidence and reflection angles of 0 degree and 18 degrees, respectively (Fig. 12). The first angle is a default position of the speaker and microphone, while the second one was chosen due to some practical
reasons as an available slight deflection of the speaker and microphone. The results were normalized to the reflection from the flat panel with the same dimensions as curved sample. Once again it is noticeable the essential disadvantage of such reflective panel’s solution. The specular reflection for an angle of 0 degree is similar for both, flat and curved samples. However, in the case of an angle equal to 18 degrees and considered high frequencies, the level of sound reflected from the sample is substantially reduced in relation to the reflection from the flat plate.

Fig. 9. Directional characteristics of sound reflection from panels shown in Fig. 5 for selected frequencies (the results obtained using finite element method).
Fig. 10. Amplitude-frequency characteristics of the sound reflection from the panels shown in Fig. 5 for the selected reflection angles (the results obtained from laboratory measurements).

Fig. 11. Diffusion coefficients $d_0$ of studied structures: the flat panel and panels with two curved edges of radius 10 cm, 20 cm, 30 cm and 40 cm.
5. Conclusions

The paper presents the research results concerning the influence of the curvature of a reflective structure on the characteristic of reflected sound. The analysis was based on some experimental studies as well as analytical and numerical calculations. First of all, it was found that all used methods clearly define the basic features of reflective structures’ responses. Next, there were compared the characteristics of sound reflections from flat, curved and semicircular elements in order to verify which panel will provide effective sound reflection and also scattering in the required band of higher frequencies, i.e. above the upper limit frequency $f_g$. The reflection from the flat panel becomes highly specular with increasing frequency of incident wave. On the other hand, the level of sound reflected from semicircular element is very low, especially in the area of specular reflections. Moreover, it is known that semicylinders might produce comb filtering that give rise to the rough sound. Thus, although they appear to be a near perfect diffuser from dispersion graphs, they do not sound like a perfect diffuser (Cox, D’ANTONIO, 2004). These interference influences are even more apparent if an array of such elements is applied. The better solution is to use the panel with curved edges which behaves like a flat panel in the specular reflection zone and like a semicircular panel outside this zone. Nevertheless, the decreasing effectiveness of the sound reflection at high frequencies induces to search for another solution of the flat panels’ narrow sound frequency range problem.

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