THE SOUND ABSORPTION PROPERTIES OF A CROSS-SHAPED ISOLATED ACOUSTIC RESONATOR

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The paper examines a new acoustic construction which differs from the classical Helmholtz resonator. The proposed acoustic resonator consists of massive plates with a rectangular-shaped slit of varied width. Sound-absorbing materials are not used in such a resonator, and the effect of absorption is achieved due to the resonance qualities of such a construction.

In a theoretical model of such construction, both the main resonance and the overtones, i.e. the odd harmonics, the impedance of the slit itself, the impedances of added air masses outside and inside the slit, the impedance of the resonator air volume, and the radiation impedance are taken into account.

On the basis of the formulas obtained, the change in the sound absorption depending on the slit width and the distance between the slit and the rigid surface is computed. It has been established that the absorption has a resonant nature and it is enhanced with the increase in the slit width. When the slit width is 65 cm, the absorption area reaches almost 4 m² at 60 Hz. The volume of the resonator is significant. When the resonator’s height is increased to 150 cm and the slit width is 30 cm, the area of absorption is as large as 3.5 m² at 35 Hz.

The dependence of the real and imaginary parts of the slit impedance and the radiation impedance on the slit width is computed, as well as the dependence of the real and imaginary parts of the resonator volume impedance on the slit width under constant distance to the rigid surface of the ceiling.

1. Introduction

Certain quantities of sound-absorbing materials must be placed in music halls for the optimization of their acoustics. These materials must have different acoustic characteristics, i.e. sound must be absorbed differently at various frequencies. This is necessary in order to obtain an optimum frequency characteristics of the hall reverberation time which is one of the most important objective criteria of the hall acoustics. The reverberation time depends on the intended purpose of the hall, its volume, the musical work being performed, the listener’s location in relation to the sound source, and the quantity and acoustic properties of the sound-absorbing materials.

The halls’ reverberation time usually requires reduction at low and medium frequencies, whereas at high frequencies the reduction is practically unnecessary because high-frequency are well-absorbed by the air itself. There are almost no materials showing both considerable absorption of low frequency energy and poor absorption of high frequency energy. Therefore special perforated acoustic structures are made employing
an air gap which is usually filled with a sound-absorbing material. Such a structure is represented by the classic Helmholtz resonator. A thin plate with varied perforation percentage is used in such a resonator with an air gap left behind it. The air gap is filled with a sound-absorbing material.

The aim of this work is to design a resonant structure that would be characterized by high absorption of low-frequency sound energy and poor absorption of the high-frequency one. The essence of the structure is that, in contrast to the Helmholtz resonator, a thick rigid plate without perforation is used, while the sound absorption effect is obtained by employing variously-shaped slits several tens centimetres wide and made of planes instead of small holes. These planes form the structure of the sound reflections which is very important for obtaining good acoustics. It is impossible to create an adequate sound reflection structure solely by using the classical Helmholtz resonant structure for the ceiling. In order to apply the proposed acoustic structure for practical purposes one must know how it absorbs sound at different frequencies and how does the absorption depend on the geometric parameters of the resonator.

2. Theory

The sound absorption properties of Helmholtz-type resonators have been investigated by many scientists in various countries. U. INGARD [1], F.P. MECHEL [2], P. GUIGNOUARD [3], B. BROUARD [4], J.F. ALLARD [5] and others have examined the changes in the sound absorption properties and impedances of thin, small-diameter, differently-shaped perforated plates made of various materials depending on the plate thickness and other parameters.

In the slit-shaped acoustic resonator, we will determine both the main resonance and the overtones, i.e. the odd harmonics, the added air masses outside and inside the slit, the energy losses in the case of a closed slit, and the effect of the resonator volume on the sound absorption.

The air gaps between the planes may be examined as holes. The calculated diagram for these gaps is shown in Figure 1.

The sound absorption of such a resonator may be calculated by the formula (1).

\[
A = 4 \rho_0 c_0 \frac{\text{Re} Z}{|Z_r + Z|^2} S_r ,
\]

(1)

where \( \rho_0 \) is the air density; \( c_0 \) is the speed of sound in the air; \( S_r \) is the area of the resonator; \( Z \) is the impedance of the hole; \( Z_r \) is the radiation impedance.

It is seen from the formula that in order to obtain the resonator’s sound absorption, the impedance of the hole itself \( Z \) and the radiation impedance \( Z_r \) must be found.

The radiation impedance of the resonator can be computed from the formula:

\[
Z_r = \rho_0 c_0 \frac{k_0^2}{4\pi^2} S_r^2 \int_0^{2\pi} d\varphi \int_0^{\pi/2+j\infty} |D(\gamma, \varphi)|^2 \sin \gamma d\gamma ,
\]

(2)

where \( k_0 \) is the wave number; \( D(\gamma, \varphi) \) is the radiation directivity pattern.
The angles of the radiation directivity pattern $\gamma$ and $\varphi$ can be determined from the diagram shown in Figure 2.

In the rectangular hole with the sides $a$ and $B$, the axis $X$ will be directed along the side $a$ and the axis $Y$ along the side $B$. Then the radiation directivity pattern will be computed by the formula:

$$D(\gamma, \varphi) = \frac{\sin[kl \cos(r, x)]}{kl \cos(r, x)} \frac{\sin[kd \cos(r, y)]}{kd \cos(r, y)},$$

where $r$ is the line connecting the origin of the coordinates with the observation point $P$. 
Now we will make the substitutions \( \cos(r, x) = \sin \gamma \cos \varphi \) and \( \cos(r, y) = \sin \gamma \sin \varphi \). Then the radiation impedance of the resonator may be written as follows:

\[
Z_r = \frac{j k_0^2}{4 \pi^2 S_r} \int_0^{2\pi} d\varphi \int_0^{\pi/2 + j \infty} d\gamma \frac{\sin^2[kl \sin \gamma \cos \varphi] \sin^2[kd \sin \gamma \sin \varphi]}{kl^2 \sin^2 \gamma \cos^2 \varphi - kd^2 \sin^2 \gamma \sin^2 \varphi} \sin \gamma d\gamma . \tag{4}
\]

Let us separate out the real part \( \Re Z_r \) and the imaginary part \( \Im Z_r \) of the impedance. The real part of the impedance can be calculated then from the formula:

\[
\Re Z_r = \frac{j k_0^2}{4 \pi^2 S_r} \int_0^{2\pi} d\varphi \int_0^{\pi/2} d\gamma \frac{\sin^2[kl \sin \gamma \cos \varphi] \sin^2[kd \sin \gamma \sin \varphi]}{kl^2 \sin^2 \gamma \cos^2 \varphi - kd^2 \sin^2 \gamma \sin^2 \varphi} \sin \gamma d\gamma , \tag{5}
\]

and the imaginary part from the formula:

\[
\Im Z_r = \frac{j k_0^2}{4 \pi^2 S_r} \int_0^{2\pi} d\varphi \int_{\pi/2 + j\infty}^{\pi/2} d\gamma \frac{\sin^2[kl \sin \gamma \cos \varphi] \sin^2[kd \sin \gamma \sin \varphi]}{kl^2 \sin^2 \gamma \cos^2 \varphi - kd^2 \sin^2 \gamma \sin^2 \varphi} \sin \gamma d\gamma . \tag{6}
\]

In order to evaluate the form of the hole, we make the following substitution: \( u = \pi/2 - j \gamma \); \( du = -j d\gamma \). Then the lower integration limit will be equal to zero and the upper one equals \(-\infty\). Thus \( \cos jx = \cos hx \). Upon integrating from 0 to \(-\infty\), the imaginary radiation impedance will be equal to:

\[
\Im Z_r = \frac{j k_0^2}{4 \pi^2 S_r} \int_0^{2\pi} d\varphi \int_{-\infty}^{\infty} d\gamma \frac{\sin^2[kl \cos \varphi \cosh u] \sin^2[kd \sin \varphi \cosh u]}{kl^2 \cos^2 \varphi \cosh^2 u - kd^2 \sin^2 \varphi \cosh^2 u} \cosh u (-j du). \tag{7}
\]

When integrated from 0 to \(\infty\), the imaginary radiation impedance is:

\[
\Im Z_r = \frac{j k_0^2}{4 \pi^2 S_r} \int_0^{2\pi} d\varphi \int_{0}^{\infty} d\gamma \frac{\sin^2[kl \cos \varphi \cosh u] \sin^2[kd \sin \varphi \cosh u]}{kl^2 \cos^2 \varphi \cosh^2 u - kd^2 \sin^2 \varphi \cosh^2 u} \cosh u du . \tag{8}
\]

This is a numerical integration procedure.

In order to calculate the sound absorption area of the resonator, one must find the impedance of the hole itself. It consists of four parts and is expressed as:

\[
Z = Z_{m0} + Z_{ma} + Z_{mi} + Z_v , \tag{9}
\]

where \( Z_{m0} \) is the impedance of the hole itself; \( Z_{ma} \) is the impedance of the added air mass outside the hole; \( Z_{mi} \) is the impedance of the added air mass inside the hole; \( Z_v \) is the impedance of the resonator volume.
The location of the impedances of the air masses added to the hole are shown in Figure 3.

The fluctuation of the air in the hole may be considered to be of plunger-type provided the velocities of the air particles are uniform over the whole area of the hole. Then the impedance of the hole itself will be equal to:

\[ Z_{m0} = \frac{\rho_0 t}{2r_0} \sqrt{\frac{8\eta}{\rho_0}} \omega + j \omega \rho_0 t \left(1 + \frac{1}{2r_0} \right) \sqrt{\frac{8\eta}{\rho_0}} \omega. \]  \hspace{1cm} (10)

The real part of this impedance determines the friction losses that arise when the air moves through the hole. Similar losses must occur on both sides of the hole. Then the impedance of the added air mass outside the hole will be equal to:

\[ Z_{ma} = Z_R + \frac{\rho_0 \Delta t_a}{2r_0} \sqrt{\frac{8\eta}{\rho_0}} \omega + j \omega \rho_0 \frac{\Delta t_a}{2r_0} \sqrt{\frac{8\eta}{\rho_0}} \omega. \] \hspace{1cm} (11)

The impedance of the added air mass inside the hole will be equal to:

\[ Z_{mi} = \frac{\rho_0 \Delta t_i}{2r_0} \sqrt{\frac{8\eta}{\rho_0}} \omega + j \omega \rho_0 \frac{\Delta t_i}{2r_0} \sqrt{\frac{8\eta}{\rho_0}} \omega. \] \hspace{1cm} (12)

The added radiation mass that is determined during the calculation of the radiation impedance \( Z_R \) must also be included. The radiation impedance \( Z_R \) is expressed in the following way:

\[ Z_R = R_r + j \omega m_r = R_r + j \omega \rho_0 r_0, \] \hspace{1cm} (13)

where \( R_r \) is the resistivity of the radiation losses.

Upon inserting the formulas (10), (11) and (12) into (9), we obtain the hole impedance:

\[ Z_m = \frac{u + (t + u/\pi)}{4S_r} \rho_0 \sqrt{\frac{8\eta}{\rho_0}} \omega + \rho_0 c_0 (k_0 a_{ef}) + \beta_1 u \]

\[ + j \omega \rho_0 \left[ t + 2\Delta t + \frac{u/t + u/\pi}{4S_r} \sqrt{\frac{8\eta}{\rho_0}} \omega \right] + Z_v, \] \hspace{1cm} (14)
where $k_0 a_{ef} = \frac{\pi}{2} k_0 d$ for the long slit of width $2d$. Then the final impedance of the long slit of width $a$ is equal to:

$$Z_m = \frac{u + (t + u/\pi)}{4S_r} \varrho_0 \sqrt{\frac{8\varrho}{\varrho_0} \omega} + \varrho_0 c_0 \left( \frac{\pi}{2} k_0 d \right) + \beta_1 u$$

$$+ j \omega \varrho_0 \left[ t + 2\Delta t + \frac{u/t + u/\pi}{4S_r} \sqrt{\frac{8\varrho}{\varrho_0} \omega} \right] + Z_v, \quad (15)$$

where $\eta$ is the air viscosity; $u$ is the slit perimeter; $\omega$ is the angular frequency; $\beta_1$ is the coefficient measuring the friction losses and $r_0$ is radius of the hole.

The impedance of the air volume of the resonator must also be determined. It can be obtained from the formula:

$$Z_v = \frac{Z_1 \coth k_0 H + \varrho_0 c_0 S_r H}{\varrho_0 c_0 \coth k_0 H + Z_1 V}, \quad (16)$$

where $Z_1$ is the impedance of the rigid surface of the ceiling; $k_0$ is the wave number; $H$ is the height of the resonator; $V$ is the volume of the resonator.

The last factor of this formula represents the relationship between the air column formed along the length of the hole and the volume of the resonator. It arises because the pressure in the air column is a variable and depends on the impedance of the added air mass inside the hole. This member characterizes the transfer of pressure to the whole volume of the resonator since the pressure undergoes changes over the whole resonator.

If the rigid surface of the ceiling is characterized by the absorption factor $\alpha$, then

$$Z_1 = Z_0 \frac{1 + \sqrt{1 - \alpha}}{1 - \sqrt{1 - \alpha}}. \quad (17)$$

The final impedance of the rectangular slit is obtained for the case when the angle of the sound wave incidence is normal. If this angle $\theta$ is not equal to $0^\circ$, the sound reflection coefficient must be computed from the formula:

$$R_{\text{refl}} = \frac{Z_0 - Z_0 \cos \theta}{Z_0 + Z_0 \cos \theta} \quad (18)$$

and from the sound absorption factor $\alpha = 1 - |R_{\text{refl}}|^2$ which depends on the angle of the sound wave incidence $\theta$. In the case of a diffusional sound field, this angle must be averaged over all angles of incidence, i.e. integrated for all angles of incidence from $0^\circ$ to $90^\circ$.

The slit may be covered with various materials with different density and air resistance. Then the loss impedance will be expressed by the formula:

$$Z_n = R_n + j Y_n, \quad (19)$$

where $R_n$ is the real part of the loss resistivity; $Y_n$ is the imaginary part of the loss resistivity.
The real part of the resistivity of losses is equal to:

\[ R_n = \rho_0 c_0 \left[ 1 + \left( \frac{\rho_0 f}{r_{\text{din}}} \right)^{-0.754} \cdot 0.0571 \right]. \tag{20} \]

The imaginary part of the resistivity of losses is equal to:

\[ Y_n = \rho_0 c_0 \cdot 0.087 \left( \frac{\rho_0 f}{r_{\text{din}}} \right)^{-0.0732} \tag{21} \]

where \( r_{\text{din}} \) is the air resistivity.

The air resistivity is characterized by the lowest frequency starting from which the loss resistivity increases as \( \sqrt{f} \).

3. Results of computations

It was established by computation how do the sound absorption area of the resonator, the imaginary parts of the slit impedance and the radiation impedance, and the real and imaginary parts of the resonator volume impedance change with changes in the slit width and the distance to the rigid surface. The thickness of the plate was chosen as 2 cm. The minimal sound absorption factors of the ceiling surface were assumed, i.e. 0.02 – 0.04 through the whole frequency range. 100 points are taken for the calculation of each curve. It is assumed in all the computations presented that the angle of incidence of the sound wave is normal.

A, m²

Fig. 4. Dependence of the sound absorption of the slit-shaped isolated acoustic resonator on the width of the slit. The distance to the rigid surface 50 cm. 1, 2, 3, 4, 5 – slit width 5, 20, 35, 50, 65 cm, respectively.
Figure 4 shows the changes taking place in the sound absorption of the resonator with changing slit width. The length and the width of the resonator were assumed to be 200 cm each, while the slit width was increased every 15 cm. The height of the resonator, i.e. the distance to the rigid surface was taken as 50 cm. A computer program has been developed that allows free variation of the geometric parameters of the resonator.

The computations show that the sound absorption is strongly dependent on the width of the slit. When the width is as small as 5 cm, the absorption value is very low too. The absorption increases rapidly with the increase in the slit width and reaches almost 4 m² at the slit width of 65 cm. The absorption has a distinct resonant character. It reaches a maximum at 40 – 50 Hz, i.e. at very low frequencies. When the width of the slit increases, the absorption and the width of the resonant curve increases too. At medium and high frequencies – starting from 350 Hz, – the absorption is lowered and repeated resonances are pronounced in this range. They are influenced by the interaction between the impedances of the slit, of the added air masses and of the resonator volume as well as by the overtones determined in the computations.

A quite different character of absorption is obtained when the height of the resonator $H$ is changed, at a remaining constant slit width. When the slit width is only 5 cm and the height changes from 5 cm to 105 cm at 25 cm intervals, the sound absorption area is as small as 0.3 m² and does not depend on the slit width. The absorption has a maximum at 30 Hz and decreases uniformly with the increase in frequency.

It is also important to know how the resonator sound absorption changes with changes in the resonator's height and volume if the slit width is kept constant. The results of computations are shown in Figure 5.

![Graph](image)

Fig. 5. Dependence of the sound absorption of a slit-shaped isolated acoustic resonator on the distance to the rigid surface. Slit width 100 cm. 1, 2, 3, 4, 5 – distance to the rigid surface 50, 75, 100, 125, 150 cm, respectively.

Sound absorption has a quite different character when the volume of the resonator is changed. At low frequencies, the absorption is practically independent of the height
of the resonator. It has a maximum at 37 Hz and achieves as much as 3.5 m\(^2\). As the frequency increases, the absorption value decreases sharply and in the remaining part of the frequency range it is determined by the repetitive air volume resonances.

The results of the computations show that an isolated acoustic resonator, whose dimensions are 200 \(\times\) 200 cm and the slit width is only 30 cm, absorbs as much as 3.5 m\(^2\) of sound energy. The resonator is made of a rigid material whose sound absorption coefficients are very small. Consequently, sound energy is only absorbed by the rectangular-shaped slit which is only 30 cm wide and 0.6 m\(^2\) in area. Thus, a slit of 0.6 m\(^2\) in area absorbs as much as 3.5 m\(^2\) of the sound energy.

The sound absorption coefficient of acoustic materials used in practice is always smaller than one. In our case the coefficient is very large – 5.8. Such coefficient is obtained under resonance only, when, in the frequency range of one to two octaves, the absorbing surface grows in area considerably and the sound absorption is therefore increased. This happens because the air velocity decreases due to friction when the air moves close to the rigid surface. The air flows around the entire plate, but the velocity of fluctuations and sound absorption are the largest in the slit; they are much larger than on the plane. When the wavelength is substantial, then the velocity of fluctuation of air particles is very low far from the slit and no sound absorption takes place on the plane. Sound is only absorbed by the environment close to the slit.

Up to the present, the sound absorption was examined in the case when the angle of incidence of the sound wave is a right one. Figures 6 and 7 show the changes in the sound absorption under other angles of incidence.

![Diagram](image)

**Fig. 6. Diagramm of the angle of incidence of the sound wave to the resonator.**

When the angle of incidence of the sound wave is small, i.e. 20°, 23° and 50°, the increase in absorption is insignificant – from 2 m\(^2\) to 2.5 m\(^2\). The resonance frequency gradually moves towards higher frequencies. When the angle of incidence is large, i.e. 65° and 80°, the absorption rises to 3 – 3.5 m\(^2\), while the resonance frequency moves to 70 and 150 Hz.

In the formulas, a distinction between the imaginary and the real parts of the impedance was made. The imaginary part of the impedance shows the reradiation energy which is equal to zero during resonance. The impedance dynamics is shown in Figure 8.

The imaginary parts of the slit increase with growing slit width and decrease along with the increase in frequency. At high frequencies, repetitive resonances are pronounced. The imaginary parts of the radiation impedance decrease as the slit width increases; they also grow along with the increase in frequency, but only at low frequencies.
Fig. 7. The relationship between the sound absorption of a slit-shaped resonator and the angle of incidence of the sound wave. Distance to the rigid surface. Slit width 30 cm. 1, 2, 3, 4, 5 – sound wave incidence angles 20°, 35°, 50°, 65°, 80°, respectively.

Fig. 8. Dependence of the imaginary part of the slit impedance and the imaginary part of the radiation impedance on the slit width in a slit-shaped isolated acoustic resonator. The distance from the slit to the rigid surface 50 cm. 1, 3, 5, 7, 9 – the imaginary parts of the slit impedance when the slit width is 5, 20, 35, 50, 65 cm, respectively. 2, 4, 6, 8, 10 – the imaginary parts of the radiation impedance when the slit width is 5, 20, 35, 50, 65 cm, respectively.

Figure 9 shows the changes in the real and imaginary parts of the resonator’s volume impedances.

Energy losses are shown by the real parts of the impedance. Their dynamics is shown in Fig. 9.
Fig. 9. Dependence of the real part of the slit impedance and the real part of the radiation impedance on the slit width in a slit-shaped isolated acoustic resonator. The distance from the slit to the rigid surface 50 cm. 1, 3, 5, 7, 9 – the real parts of the slit impedance when the slit width is 10, 30, 50, 70, 90 cm, respectively. 2, 4, 6, 8, 10 – the real parts of the radiation impedance when the slit width is 10, 30, 50, 70, 90 cm, respectively.

The real part of the slit impedance is independent of the slit width and frequency, while the real part of the radiation impedance increases with the increase in frequency.

Fig. 10. Dependence of the real and imaginary parts of the volume impedance on the slit width in a slit-shaped isolated acoustic resonator. The distance to the rigid surface 50 cm. 1, 3, 5, 7, 9 – the real parts of the volume impedance when the slit width is 5, 20, 35, 50, 65 cm, respectively. 2, 4, 6, 8, 10 – the imaginary parts of the volume impedance when the slit width is 5, 20, 35, 50, 65 cm, respectively.
but only up to medium frequencies. As the slit width increases, the real part of the impedance increases too.

The resonator's sound absorption is greatly influenced by its volume. The changes in the imaginary and the real parts of the volume impedance are shown in Figure 10.

The increases in the imaginary and the real parts of the volume impedance produced by the increase in frequency and slit width are almost the same. The impedances are influenced by the elasticity of the resonator's air which is determined by the volume and height of the closed air mass. The fluctuating air mass is determined by the air in the slit and the added air mass, which is given by the imaginary part of the radiation impedance. When the air elasticity is equal to the fluctuating air mass, the resonance takes place.

4. Conclusions

1. A large isolated resonator with a rectangularly-shaped slit absorbs sound energy well at low frequencies. The sound absorption increases rapidly with the increase in the slit width, while the frequency at which the maximum absorption is reached changes a little. Greater sound absorption is obtained when the height of the resonator, but not the slit width is increased.

2. The real parts of the slit impedance depend on the slit width and frequency, whereas the imaginary parts are little frequency-dependent. The imaginary and the real parts of the resonator's volume impedance depend only on the frequency and the slit width up to medium frequencies. As frequency increases the impedances are determined by repetitive resonances.

References


