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ACOUSTIC STREAMING INDUCED BY THE NON-PERIODIC SOUND IN A VISCOUS MEDIUM

Anna PERELOMOVA

Gdańsk University of Technology
Faculty of Applied Physics and Mathematics
G. Narutowicza 11/12, 80-952 Gdańsk, Poland
e-mail: anpe@mifgate.pg.gda.pl

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Instantaneous radiation force of acoustic streaming in a thermoviscous fluid is the subject of investigation. Dynamic equation governing the velocity of acoustic streaming is a result of splitting of the conservative differential equations in partial derivatives basing on the features of all possible types of a fluid motion. The procedure of deriving does not need averaging over sound period. It is shown that the radiation force consists of three parts, one corresponding to the classic result (while averaged over sound period), the second being a small negative term caused by diffraction, and the third one. This last term equals exactly zero for periodic sound (after averaging) and differs from zero for other types of sound. Sound itself must satisfy the well-known Khokhlov–Zabolotskaya–Kuznetsov equation describing the weakly diffracting nonlinear acoustic beam propagating over viscous thermconducting fluid. The parts of radiation acoustic force, relating to the sound non-periodicity and diffraction, are discussed and illustrated.

Keywords: acoustic streaming, acoustic radiation force, nonlinear hydrodynamics.

1. Origins of acoustic streaming and general remarks about theory

Acoustic streaming is the bulk vortex movement of fluid following the acoustic wave. The reason of acoustic streaming is loss in acoustic momentum. It is well understood that the origins of acoustic streaming are jointly nonlinearity and thermoviscosity of the flow [1, 2]. In the last decades, many new applications of acoustic streaming in technique, medicine and biology appeared [3, 4]. That needs to develop theory of acoustic streaming relatively to pulsed and non-periodic sound. Traditional theory deals with strictly periodic sound in a role of origin of acoustic streaming. That permits to derive governing equation for vortex flow by simple averaging of momentum and continuity equations over sound period. The important inconsistency of classical treatment are, among other, supposing that the fluid is incompressible and discarding equation of energy balance [1, 2]. The present study continues investigations of acoustic streaming and heating basing on consistent division of conservative equations into specific parts [5, 6]. The radiation acoustic force caused by both periodic and non-periodic sound is derived in the form easy for finding out effects connected with non-periodicity and diffraction of sound. The examples of radiation force in a weakly diffracting acoustic beam with slowly varying envelope in water for frequency of carrier $f = 5$ MHz and transducer radius $R = 1$ cm are presented.

2. The instantaneous dynamic equation of acoustic streaming

The basic idea which was worked out by the author is to combine the momentum, mass and energy equations in the differential form basing on the specific features of all possible types of motion, both quick acoustic, and slow vortex and entropy ones. On the other hand, the determination of these types of motion also follows from the linearized Navier–Stockes equations, and therefore depends on flow geometry, external forces and presence of boundaries. For every type of motion, there are independent on time links of two thermodynamic variables (for example, excess density and pressure) and components of particles velocity [5, 6]. That permits to use only one reference variable for every mode. The linear combining of equations is proceeded in order to derive dynamic equations in reference variable for every mode by means of excluding all other in the linear part. The nonlinear terms are responsible for different interactions of modes.

In the simplest case of flow over uniform initially background, links of velocity components $v_{x,a}$, $v_{y,a}$, $v_{z,a}$ and excess pressure p_a and density ρ_a in the progressive in the positive direction of axis y acoustic beam are as follows:

$$\psi_a = \begin{pmatrix} v_{x,a} \\ v_{y,a} \\ v_{z,a} \\ p_a \\ \rho_a \end{pmatrix} = \begin{pmatrix} \dfrac{\sqrt{\mu}c_0}{\rho_0}\dfrac{\partial}{\partial x}\displaystyle\int \mathrm{d}y \\ \dfrac{c_0}{\rho_0}\left(1 - \sqrt{\mu}\Delta_\perp \displaystyle\int \mathrm{d}y \int \mathrm{d}y\right) - \dfrac{b}{2\rho_0^2}\dfrac{\partial}{\partial y} \\ \dfrac{\sqrt{\mu}c_0}{\rho_0}\dfrac{\partial}{\partial z}\displaystyle\int \mathrm{d}y \\ c_0^2 + \dfrac{\delta c_0}{\rho_0}\dfrac{\partial}{\partial y} \\ 1 \end{pmatrix} \rho_a\,, \qquad (1)$$

where $\sqrt{\mu} = (kR)^{-1}$ is a small parameter expressing the ratio of longitudinal and transversal scales of perturbations (k, R denote sound longitudinal wavenumber and the radius of transducer), ρ_0 is unperturbed background density, and c_0 is the velocity of infinitely small amplitude sound, $\Delta_\perp$ is a Laplacian operating in the plane (x, z). Thermal and total attenuations are marked as follows: $\delta = \chi\left(\dfrac{1}{C_V} - \dfrac{1}{C_P}\right)$, $b = 4\eta/3 + \eta_B + \delta$, where χ, η, η_B mark thermal conduction and shear and bulk viscosities, C_V, C_P

are heat capacities under constant volume and pressure, respectively. The both branches of vortex flow are solenoidal ($\boldsymbol{\nabla}\mathbf{v}_{\mathrm{vortex}} = 0$):

$$\psi_{\mathrm{vortex}} = \begin{pmatrix} v_{x,\,\mathrm{vort}} \\ v_{y,\,\mathrm{vort}} \\ v_{z,\,\mathrm{vort}} \\ p_{\mathrm{vort}} \\ \rho_{\mathrm{vort}} \end{pmatrix} = \begin{pmatrix} -\partial/\partial y \\ \sqrt{\mu}\partial/\partial x \\ 0 \\ 0 \\ 0 \end{pmatrix} \varphi_1 + \begin{pmatrix} 0 \\ \sqrt{\mu}\partial/\partial z \\ -\partial/\partial y \\ 0 \\ 0 \end{pmatrix} \varphi_2 \,. \tag{2}$$

Linear combining of conservation equations valid within accuracy of order M^2, μM, $bM/(\rho_0 c_0 \lambda)$, $b\mu/(\rho_0 c_0 \lambda)$, where M is the Mach number, with account for specific features of modes leads, among other, to equation analogous to the well-known Khokhlov–Zabolotskaya–Kuznetsov one (B/A is the parameter of nonlinearity):

$$\begin{aligned} &\frac{\partial}{\partial t} p_a + c_0 \left(\frac{\partial}{\partial y} p_a + \frac{\mu}{2} \int \Delta_\perp p_a \, \mathrm{d}y \right) - \frac{b}{\rho_0} \frac{\partial^2}{\partial y^2} p_a \\ &\qquad + \frac{1}{\rho_0 c_0} \left(1 + \frac{B}{2A} \right) p_a \frac{\partial}{\partial y} p_a = 0, \end{aligned} \tag{3}$$

and to the following equation governing the vortex flow if rightwards sound beam is dominative (written on for longitudinal component of particle velocity):

$$\frac{\partial}{\partial t} v_{y,\mathrm{vort}} - \frac{\eta}{\rho_0} \frac{\partial^2}{\partial y^2} v_{y,\mathrm{vort}} + (\mathbf{v}_{\mathrm{vort}} \boldsymbol{\nabla})\, v_{y,\mathrm{vort}} = F_y = F_{y,\,\mathrm{class}} + F_{y,p} + F_{y,\mu} \,, \tag{4}$$

$$\begin{aligned} F_{y,\,\mathrm{class}} &= -\frac{b}{\rho_0^3 c_0^5} p_a \frac{\partial^2 p_a}{\partial t^2} \,, \\ F_{y,p} &= \frac{b}{\rho_0^3 c_0^5} \frac{\partial^2}{\partial t^2} \int \mathrm{d}y \int \mathrm{d}y \left(\frac{1}{2} p_a \frac{\partial^2}{\partial y^2} p_a + \frac{3\mu}{4} \left(\frac{\partial p_a}{\partial r} \right)^2 \right) \\ &\quad + \frac{b}{\rho_0^3 c_0^7} \left(2c_0^2 - \frac{\partial^2}{\partial t^2} \int \mathrm{d}y \int \mathrm{d}y \right) \\ &\quad \cdot \left(\frac{3}{4} \frac{\partial^2 p_a^2}{\partial t^2} - \frac{\partial^2}{\partial t^2} \int \mathrm{d}y \int \mathrm{d}y \cdot p_a \frac{\partial^2}{\partial y^2} p_a + \frac{3c_0}{2} \frac{\partial}{\partial t} \int \mathrm{d}y \left(\frac{\partial p_a}{\partial y} \right)^2 \right), \\ F_{y,\mu} &= \frac{b\mu}{\rho_0^3 c_0^3} \left(p_a \Delta_\perp p_a - \frac{3}{2} \left(\frac{\partial p_a}{\partial r} \right)^2 \right). \end{aligned}$$

For simplicity, the cylindrical symmetry is supposed: $\Delta_\perp = (1/r)\partial/\partial r + \partial^2/\partial r^2$, $r = \sqrt{x^2 + z^2}$. Radiation force in the right-hand side of Eq. (4) consists, therefore, of three parts. The first $F_{y,\,\mathrm{class}}$ after averaging over sound period corresponds to the famous classic term, the second one $F_{y,p}$ would result in exactly zero for strictly periodic sound while averaged over sound period (that is not valid for any non-periodic sound), and the last $F_{y,\mu}$ is a small term proportional to diffraction.

3. Examples of radiation force produced by quasi-periodic sound

The advantage in theory is that the radiation force is instantaneous, because it is a result of linear combining of conservation equations, which does not need averaging over sound period. Its form (4) provides easy examining of difference in radiation force produced by quasi-periodic sound comparatively to the strictly periodic one.

In view of complexity of joint solution of equations (3) and (4), any simple waveform in the role of acoustic source is studied. As an example, the sound with slowly varying envelope is taken in the form:

$$p_a = \frac{P_0}{2} F(\Omega(t - y/c_0)) \left(-\frac{i\,\mathrm{Exp}\left(-\frac{r^2}{(1-2iy/L)R^2} + i\omega(t - y/c_0) \right)}{1 - 2iy/L} \right) + cc, \qquad (5)$$

where $F(\Omega(t - y/c_0))$ is an envelope, $L = \omega R^2/2c_0 = R/(2\mu)$ denotes the diffraction length. Carrying sound frequency is much larger than that of envelope: $\omega \gg \Omega$. The waveform (5) satisfies Eq. (3) in the linear non-viscous limit $(b = 0)$ with accuracy of order $\mathrm{Max}(\Omega/\omega, \mu^2)$. In the spatial domain closer to the transducer than the diffraction length, $y < L$, and for the envelope in the form $F(\Omega(t - y/c_0)) = \mathrm{Exp}(-(\Omega(t - y/c_0))^2)$, the acoustic pressure may be approximated by the following formula:

$$p_a = P_0\,\mathrm{Exp}\left(-(\Omega(t - y/c_0))^2\right) \mathrm{Exp}(-r^2/R^2) \sin(\omega(t - y/c_0)). \qquad (6)$$

Results of calculations are presented below for values widely used in experiments for water [7, 8]: $f = \omega/2\pi = 5$ MHz, $R = 1$ cm. Averaged over period of carrier dimensionless parts of radiation force are shown in Figs. 1, 2.

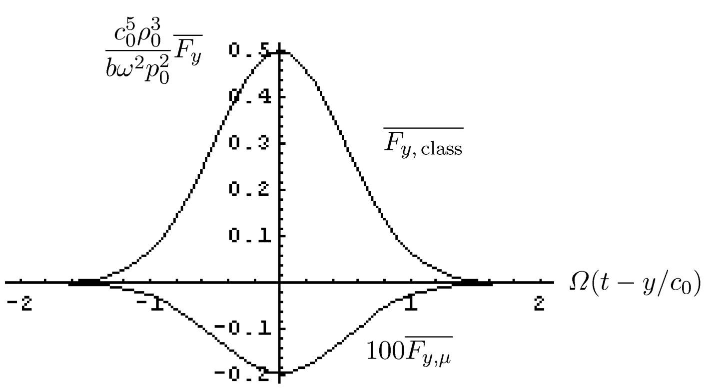

Fig. 1. The averaged over period of carrier sound dimensionless parts of radiation acoustic force correspondent to $\overline{F_{y,\,\mathrm{class}}}$ and $\overline{F_{y,\mu}}$ versus dimensionless retarded time.

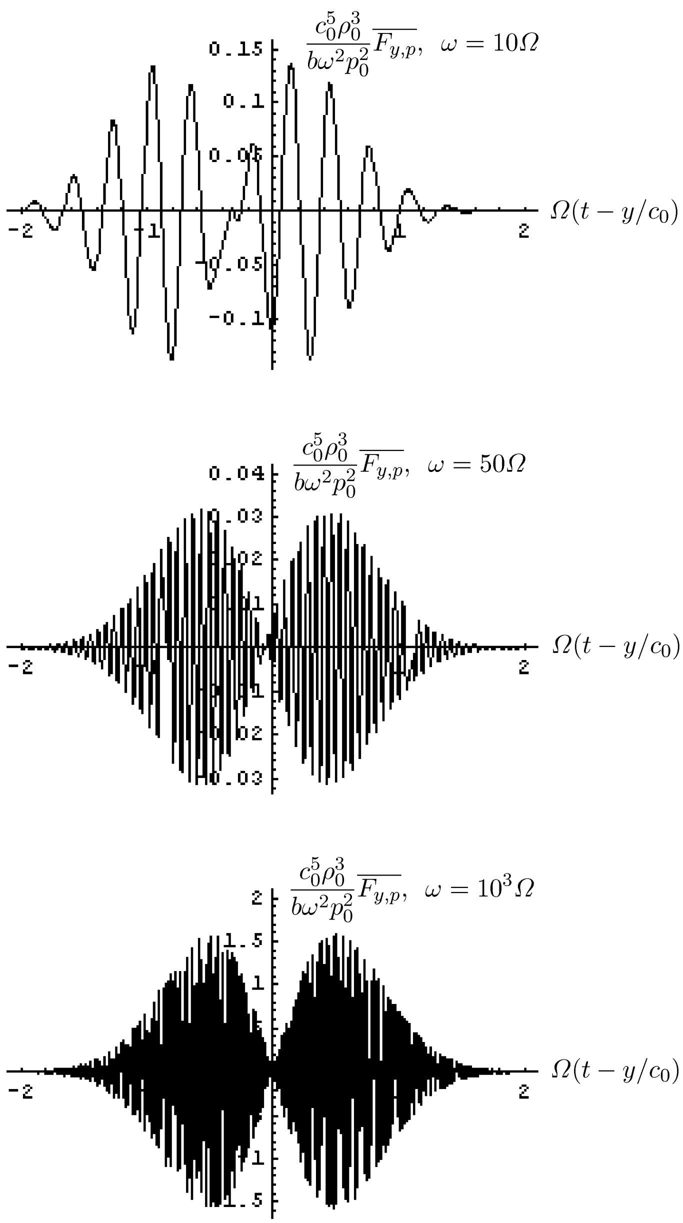

Fig. 2. The averaged over period of carrier sound dimensionless part of radiation acoustic force correspondent to $\overline{F_{y,p}}$ versus dimensionless retarded time for different ω/Ω: 10, 50, 1000.

4. Discussion and conclusions

The greatest term is that one following from the classic theory, $\overline{F_{y,\,\mathrm{class}}}$ (over lining denotes average over carrier period $T = 2\pi/\omega$). The additional term caused by diffraction, $\overline{F_{y,\mu}}$, is proportional to the diffraction parameter μ, and in typical conditions of experiments in water achieves 4% of $\overline{F_{y,\,\mathrm{class}}}$. That may explain lower experimental values comparatively to the theory predictions [7, 9]. The last part originating from the non-periodicity, $\overline{F_{y,p}}$, strongly depends on ratio of carrier and envelope frequencies, ω/Ω. The relatively smaller quantities of Ω would result to the greater quantities of $\overline{F_{y,p}}$: for $\omega/\Omega = 10$, the ratio of magnitudes of $\overline{F_{y,p}}$ and $\overline{F_{y,\,\mathrm{class}}}$ achieves 20%.

In this study, only the radiation force for sound of every type, not certainly periodic, is paid attention to. The solution of governing equation (4) is in general very complex problem, not only in view of requirement of accurate solution of the KZK equation (3) providing the correct radiation force. It is well-understood that, among of viscosity, nonlinear convective term should be necessarily taken into account. It is a reason to streaming not to grow infinitively.

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