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MAGNETOMECHANICAL COUPLING IN TRANSDUCERS

ZBIGNIEW KACZKOWSKI

Institute of Physics, Polish Academy of Sciences
(02-668 Warsaw, Al. Lotników 32/46)

The basic definitions of the coefficient of magnetomechanical coupling, k, based on generalized definitions of magnetic, mechanical (elastic) and piezomagnetic energies, and on the magnetic quantities in the case of transmitting transducers and the mechanical quantities in the case of receiving transducers, i.e. for the mechanomagnetic coefficient, are presented. The relations between the coupling coefficient and the coefficients occurring in each system of piezomagnetic equations and for different systems of equations and systems of units, and also for induction and magnetization, are derived.

1. Initial remarks

The properties of piezomagnetic (magnetostrictive) materials are characterized by piezomagnetic coefficients occurring in piezomagnetic equations [1-12, 14, 15] and by other basic quantities and physical parameters, such as e.g. the induction (magnetization M, I, J) of saturation B_s, the magnetostriction of saturation λ_s, the magnetocrystalline anisotropy constants $K_1, K_2, K_3, \ldots$, the density ϱ, the resistivity ϱ_{el}, the sound velocities and their temperature dependencies, the Curie temperature T_C or the Néel temperature T_N, the mechanical strength etc. [1-5, 8, 10-12, 14-15].

Of piezomagnetic coefficients occurring in piezomagnetic equations, such mechanical coefficients can be distinguished as the coefficients c or the moduli (s, E, G) of elasticity defined for the constant field intensity H or the constant magnetization (M, I, J), or the induction B, e.g. s_H, E_H, E_J, E_B; the magnetic coefficients (of permeability μ or of susceptibility $\varkappa$, or their reciprocals) defined for the constant mechanical stress T, σ, τ (of a free sample), or for the constant strains S, ε, λ, e.g. $\mu_T, \mu_S, \mu_\lambda, \varkappa_\sigma, \varkappa_\varepsilon$; and the magnetomechanical coefficients such as the piezomagnetic sensitivity d or λ, the constant (coefficient) of magnetostriction h, λ or Γ, the coefficient of field stresses e and the coefficient of inductive strains g, e.g. [1-12, 14, 15]. Although each of these coefficients describes some property of piezomagnetic material as a function of magnetization, amplitude, or temperature, but none of them is itself a sufficient parameter for the usefulness of the piezomagnetic material to be satisfactorily determined. It is only the set of piezomagnetic coefficients occurring in at least one of the systems of piezomagnetic equations, i.e. at least 3 coefficients, e.g. E_H, μ_T and d, that gives almost enough information. None of them, however, can be a measure of the effectiveness of energy conversion.

This function is fulfilled by the coefficient of magnetomechanical coupling, k, which in addition to the mechanical and electrical quality factor, the electroacoustical efficiency, the vibration amplitude etc., permits the comparison of the properties of piezotronic materials and piezomagnetic transducers [1, 10-12, 14]. Its analogue in piezoelectric materials is the coefficient of electromechanical coupling [1, 10-12, 14].

Part of the energy supplied to the transmitting transducer is converted to the energy of elastic oscillation and radiated into the medium loading the transducer. The opposite process occurs in the receiving transducer. The mechanical energy E_e (the acoustic energy E_a), supplied to the receiver, is partly converted to the magnetic energy E_m (the electrical energy E_{el}). The measure of this transformation is the coupling coefficient: that of magnetomechanical coupling in the case of piezomagnetic materials and transducers, and that of electromechanical coupling in the case of piezoelectrical materials and transducers, e.g. [1, 14]. The coefficient of optomechanical coupling can be defined similarly in the case of piezooptic materials and transducers etc.

2. Definition of coupling

2.1. *Coupled electrical circuits*

Coupled electrical circuits are the systems permitting the transmission of the *ac* power from one circuit to another.

The quantity characterizing quantitatively the degree of coupling is the coupling coefficient which is in direct proportion to power transmitted, and in inverse proportion to the geometrical mean of the power of the circuits,

$$k = \frac{W_p}{\sqrt{W_1 W_2}} = \frac{\hat{Z}_{sp}}{\sqrt{\hat{Z}_1 \hat{Z}_2}}, \tag{1}$$

where W_1 and W_2 are the energies of the primary and the secondary circuits, respectively; W_p is the transmitted energy, $\hat{Z}_{sp}$, $\hat{Z}_1$ and $\hat{Z}_2$ are the characteristic impedances of the coupling element and the primary and the secondary circuits, respectively. As in the theory of electrical circuits, the definition of coupling is derived in the case of piezotronic materials and components.

2.2. *Generalized definitions of energies related to the coupling*

The part of energy which is transformed to another energy in magneto-electro- or opto-mechanical transformations, can be called the piezo-magnetic, piezo-electric, or piezo-optic energy, W_p.

The proper energies (energy densities), which participate in conversion in the case of piezomagnetic transducers, i.e. the magnetic energy W_m, the mechanical (elastic) energy W_e and the piezomagnetic energy W_p, can be defined in the following way:

$$W_m = \frac{1}{2}\sum_{i=1}^{i=3}\sum_{j=1}^{j=3} B_i H_j = \frac{1}{2}\sum_{i=1}^{i=3}\sum_{j=1}^{j=3} \mu_{Tij} H_i H_j, \quad (2)$$

$$W_e = \frac{1}{2}\sum_{p=1}^{p=6}\sum_{q=1}^{q=6} S_p T_q = \frac{1}{2}\sum_{p=1}^{p=6}\sum_{q=1}^{q=6} s_{Hpq} T_p T_q = \frac{1}{2}\sum_{p=1}^{p=6}\sum_{p=1}^{q=6} \frac{1}{c_{Hpq}} T_p T_q, \quad (3)$$

$$W_p = \frac{1}{2}\sum_{i=1}^{i=3}\sum_{q=1}^{q=6} d_{iq} H_i T_q, \quad (4)$$

where B_i, H_i and H_j are respectively 3 components of the axial vectors of the induction B and the field intensity H; S_p, T_p and T_q are respectively 6 components of the strain tensors S_{ij} and the mechanical stresses T_{ij}; μ_{Tij} are 9 components of the tensor of magnetic permeability at constant stresses (of a free sample); s_{Hpq} are 36 components of the tensor of the elastic coefficient at constant magnetic field, i.e. the proper state corresponding in the extreme to an open electrical circuit, which is in practice a system of constant current efficiency, i.e. very high impedance; and d_{iq} are 18 components of the pseudotensor of piezomagnetic sensitivity. The differential signs were neglected in all these quantities, but in should be borne in mind that these relations are valid over the range of small reversible lossless linear variations of the quantities discussed above.

2.3. *Components of magnetic, mechanical and piezomagnetic vectors and tensors*

The components of the vectors and tensors occurring in equations (2)-(4) in the most general expansion before simplification will have the following form,

$$B = B_i \quad (i = 1, 2, 3 \text{ or } x, y, z), \quad (5)$$

$$H = H_j \quad (j = 1, 2, 3 \text{ or } x, y, z), \quad (6)$$

$$S = S_{ij} \quad (i, j = 1, 2, 3 \text{ or } x, y, z), \quad (7)$$

$$T = T_{ij} \quad (i, j = 1, 2, 3 \text{ or } x, y, z), \quad (8)$$

$$\mu_T = [\mu_{Tij}] = \begin{bmatrix} \mu_{T11}, \mu_{T12}, \mu_{T13} \\ \mu_{T21}, \mu_{T22}, \mu_{T23} \\ \mu_{T31}, \mu_{T32}, \mu_{T33} \end{bmatrix} = \begin{bmatrix} \mu_0(1+\varkappa_{T11}), \mu_0\varkappa_{T12}, \mu_0\varkappa_{T13} \\ \mu_0\varkappa_{T21}, \mu_0(1+\varkappa_{T22}), \mu_9\varkappa_{T23} \\ \mu_0\varkappa_{T31}, \mu_0\varkappa_{T32}, \mu_0(1+\varkappa_{T33}) \end{bmatrix}, \tag{9}$$

$$d = d_{ijk} \quad (i, j, k = 1, 2, 3), \tag{10}$$

$$c = c_{Hijkl} \quad (i, j, k, l = 1, 2, 3), \tag{11}$$

$$s = s_{Hijkl} \quad (i, j, k, l = 1, 2, 3). \tag{12}$$

2.4. *Magnetic energy*

The energy stored in free material in the course of magnetization can be defined on the basis of work put in its magnetization. The increases in energy of a free unit volume (in the case of isothermal and reversible transformation) can be defined by the following relations:

$$dW_m = HdB_T = \mu_0 Hd(H+M_T) = \mu_0(1+\varkappa_T)HdH = \mu_T HdH \tag{13}$$

or

$$dW_m = H_i dB_{Ti} = \mu_{Tij} H_i dH_j, \tag{14}$$

which after expansion gives

$$dW_m = \mu_{T11}H_1dH_1 + \mu_{T12}H_1dH_2 + \mu_{T12}H_1dH_3 + \mu_{T21}H_2dH_1 + \\ + \mu_{T22}H_2dH_2 + \mu_{T23}H_2dH_3 + \mu_{T31}H_3dH_1 + \mu_{T32}H_3dH_2 + \mu_{T33}H_3dH_3. \tag{15}$$

Symmetricity of the tensor μ_{ij} can be proved in the following way. The differentials of the increases in energy with respect to the components of the fields H_1 and H_2 have the following form

$$\frac{\partial W_m}{\partial H_1} = \mu_{T11}H_1 + \mu_{T21}H_2 + \mu_{T31}H_3, \tag{16}$$

$$\frac{\partial W_m}{\partial H_2} = \mu_{T12}H_1 + \mu_{T22}H_2 + \mu_{T32}H_3. \tag{17}$$

Differentiation of equation (16) with respect to H_2 and that of (17) with respect to H_1 give

$$\frac{\partial}{\partial H_2}\left(\frac{\partial W_m}{\partial H_1}\right) = \mu_{T21} = \frac{\partial^2 W_m}{\partial H_1 \partial H_2}, \tag{18}$$

$$\frac{\partial}{\partial H_1}\left(\frac{\partial W_m}{\partial H_2}\right) = \mu_{T12} = \frac{\partial^2 W_m}{\partial H_1 \partial H_2}, \tag{19}$$

and thus

$$\mu_{T12} = \mu_{T21}, \tag{20}$$

and it can be shown similarly that

$$\mu_{T13} = \mu_{T31}, \tag{21}$$

$$\mu_{T23} = \mu_{T32}, \tag{22}$$

i.e. the tensor μ_{Tij} is symmetrical.

After integration of equation (15) and consideration of relations (20)-(22), the proper magnetization energy is

$$W_m = \frac{1}{2}\mu_{T11}H_1^2 + \mu_{T12}H_1H_2 + \mu_{T13}H_1H_3 + \frac{1}{2}\mu_{T22}H_2^2 + \mu_{T23}H_2H_3 + \frac{1}{2}\mu_{T33}H_3^2. \tag{23}$$

Using the Einstein summation notation saying that when a letter index occurs twice in the same term, summation should be done with respect to it, relations (2) and (23) can be written in the following way

$$W_m = \frac{1}{2}\mu_{Tij}H_iH_j \quad (i, j = 1, 2, 3). \tag{24}$$

2.5. *Elastic strain energy*

The change in the energy of elastic strain of a material unit volume, W_e, after consideration of the Hooke law, is defined by the following relations

$$dW_e = T_{ij}dS_{Hij} = s_{Hijkl}T_{ij}dT_{kl} \quad (i, j, k, l = 1, 2, 3). \tag{25}$$

In equation (25), 81 components occur in the most general form. In specific cases some simplifications can be introduced and in the extreme case of strains, e.g. longitudinal linear strains, 1 elastic coefficient, known as the Young's modulus Y_H or E_H, occurs. Similarly to the magnetic energy case for the symmetry condition, the number of different elastic coefficients may be reduced from 81 up to the maximum 36 [13, 16]. The components of the stress tensor, T_{ij}, define the forces acting on the elementary surfaces of an elementary solid cube. The first index: 1, 2, 3 or x, y, z denote the direction normal to the surface considered, e.g. x normal to the yz surface, while the second index denotes the direction of the stress component. With absent bulk forces and bulk torsional moments, and with uniform stresses, for the state of static equilibrium the tensor is symmetrical and has 6 components, i.e.

$$T_{ij} = T_{ji} \quad (i, j = 1, 2, 3). \tag{26}$$

The tensor of the strains S_{ij} is similarly symmetrical, i.e.

$$S_{ij} = S_{ji} \quad (i, j = 1, 2, 3). \tag{27}$$

The symmetry of elastic coefficients follows from the symmetry of the strain and stress tensors

$$c_{Hijkl} = c_{Hjikl} = c_{Hijlk} \quad (i, j, k, l = 1, 2, 3). \tag{28}$$

For an elastic medium, the strain energy is a function of state, independent of intermediate states, and then

$$c_{Hijkl} = c_{Hklij}. \tag{29}$$

The Hooke's law in tensor notation has the following forms

$$T_{ij} = c_{Hijkl} S_{kl} \quad (i, j, k, l = 1, 2, 3). \tag{30}$$

$$S_{ij} = s_{Hijkl} T_{kl} \quad (i, j, k, l = 1, 2, 3). \tag{31}$$

The flexibility or the coefficient (constant) of elasticity, c_{Hijkl}, is the inverse elastic modulus s_{Hijkl} and

$$s_{Hijkl} = s_{Hjikl} = s_{Hijlk} = s_{Hklij}, (i, j, k, l = 1, 2, 3). \tag{32}$$

The symmetry of the coefficients with respect to the first or second pair of indices permits a simpler matrix form to be used. Two-digit tensor notation can be replaced with single-digit matrix form, e.g. [13, 16], according to the following principle:

$$11 \to 1,\ 22 \to 2,\ 33 \to 3,\ 23 \to 4,\ 13 \to 5,\ 32 \to 4,\ 31 \to 5,\ 12 \to 6,\ 21 \to 6,$$

i.e. for example,

$$[T_{p.}] = [T_1, T_2, T_3, T_4, T_5, T_6], \quad [S_q] = [S_1, S_2, S_3, S_4, S_5, S_6],$$

and

$$s'_{Hpq} = s_{Hijkl} \begin{cases} p = i, & \text{when } i = j,\ p = m+3, \quad \text{when } i \neq j \neq m \\ & (i, j, k, l = 1, 2, 3), \\ q = k, & \text{when } k = l,\ q = m+3, \quad \text{when } k \neq l \neq m \\ & (p, q = 1, 2, 3, \ldots, 6), \end{cases} \tag{33}$$

and in view of the symmetry of strains it can be assumed that

$$2s'_{Hij} = s_{Hij}, \quad \text{when } i \neq j = 1, 2, 3, \tag{34}$$

and the dash can be neglected in s_{Hii}, i.e.

$$s'_{Hij} = s_{Hij}, \quad \text{when } i = j = 1, 2, 3. \tag{35}$$

After the above simplifications in notation, integration, reduction and ordering, equation (25) for the proper energy of elastic strains becomes

$$\begin{aligned} W_e = \frac{1}{2} s_{H11} T_1^2 + s_{H12} T_1 T_2 + s_{H13} T_1 T_3 + s_{H14} T_1 T_4 + s_{H15} T_1 T_5 + \\ + s_{H16} T_1 T_6 + \frac{1}{2} s_{H22} T_2^2 + s_{H23} T_2 T_3 + s_{H24} T_2 T_4 + s_{H25} T_2 T_5 + s_{H26} T_2 T_6 + \\ + \frac{1}{2} s_{H33} T_3^2 + s_{H34} T_3 T_4 + s_{H35} T_3 T_4 + s_{H36} T_3 T_6 + \frac{1}{2} s_{H44} T_4^2 + s_{H45} T_4 T_5 + \\ + s_{H46} T_4 T_6 + \frac{1}{2} s_{H55} T_5^2 + s_{H56} T_5 T_6 + \frac{1}{2} s_{H66} T_6^2, \end{aligned} \tag{36}$$

which in shorter notation gives the form

$$W_e = \frac{1}{2} s_{Hpq} T_p T_q \quad (p, q = 1, 2, 3, 4, 5, 6), \tag{37}$$

i.e. formula (3) in the Einstein notation.

2.7. *Piezomagnetic energy*

The piezomagnetic energy of a transducer is the part of magnetic or mechanical energy which is transformed into other energy. The proper piezomagnetic energy is characterized, for example, as the product of additional strain caused by the intensity of the magnetic field H_k

$$S_i = d_{ijk} H_k \quad (i, j, k = 1, 2, 3) \tag{38}$$

and the mechanical stresses T_{ij}

$$W_p = \frac{1}{2} d_{ijk} T_{ij} H_k \tag{39}$$

or the additional induction B_i (magnetization M_i) caused by the external stresses T_{ij}

$$B_i = d_{ijk} T_{jk} \quad (i, j, k = 1, 2, 3), \tag{40}$$

and the field intensity H_i, i.e. again

$$W_p = \frac{1}{2} d_{ijk} H_i T_{jk}. \tag{41}$$

In the case of the symmetrical tensor T_{jk} (with the absence of the internal moments of a force), the piezomagnetic sensitivity, which functions here as the proportionality coefficient, is symmetrical, i.e.

$$d_{ijk} = d_{ikj} \tag{42}$$

and the number of its independent components reduces from 27 (see formula (10)) to 18.

After consideration of (42), formulae (39) or (41) in the expanded form are the following

$$\begin{aligned} W_p = & \frac{1}{2} d_{111} H_1 T_{11} + d_{112} H_1 T_{12} + d_{113} H_1 T_{13} + \frac{1}{2} d_{122} H_1 T_{22} + d_{123} H_1 T_{23} + \\ & + \frac{1}{2} d_{133} H_1 T_{33} + \frac{1}{2} d_{211} H_2 T_{11} + d_{212} H_2 T_{12} + d_{213} H_2 T_{13} + \frac{1}{2} d_{222} H_2 T_{22} + \\ & + d_{223} H_2 T_{23} + \frac{1}{2} d_{233} H_2 T_{33} + \frac{1}{2} d_{311} H_3 T_{11} + d_{312} H_3 T_{12} + d_{313} H_3 T_{13} + \\ & + \frac{1}{2} d_{322} H_3 T_{22} + d_{323} H_3 T_{23} + \frac{1}{2} d_{333} H_3 T_{33}. \end{aligned} \tag{43}$$

Substitution of the single-digit tensor notation for the two-digit form leads to 6 components of the stress tensor T_q and 18 components of the tensor of piezomagnetic sensitivity, d_{iq},

$$W_p = \frac{1}{2} d_{iq} H_i T_q, \tag{44}$$

i.e. formula (4) in the Einstein notation.

2.8. Definition of the coefficient of magnetomechanical coupling

The magnitude of magnetomechanical coupling depends on the products of the density ratios of the energy transformed to the supplied. As in the theory of circuits, the coefficient of magnetomechanical coupling, k, is defined as the ratio of the piezomagnetic (coupling) energy to the geometrical mean of magnetic and mechanical energy,

$$k = \frac{W_p}{\sqrt{W_m W_e}}, \tag{45}$$

which after insertion of relations (2)-(4), or simpler relations (24), (37) and (44), and squaring, gives

$$k^2 = \frac{(d_{iq} H_i T_q)^2}{\mu_{Tij} H_i H_j s_{Hpq} T_p T_q}. \tag{46}$$

For simple longitudinal or radial vibration, most components vanish and only one remains, e.g. k_{33}. Then indices and summations can be neglected, and

$$k^2 = \frac{d^2 H^2 T^2}{\mu_T H^2 s_H T^2} = \frac{d^2}{\mu_T s_H} = \frac{d^2 E_H}{\mu_T}. \tag{47}$$

3. Coefficients of magnetomechanical and mechanomagnetic coupling

3.1. Coefficients of magnetomechanical coupling

The coupling coefficient, defined in the previous section, is universal, since it applies both to magnetomechanical transducers, or magnetoacoustic transducers transforming magnetic energy into mechanical or acoustic energy, and to mechanomagnetic (acoustomagnetic) transducers under the assumption of idealized states, i.e. when no losses occur, for example, in magnetic, magnetomechanical, or mechanical energy, and of reversible behaviour. Under this assumption the direction of change is not specified and the definition is thus universal.

The coefficient of magnetomechanical coupling may be compared with the coefficient of electromechanical coupling which occurs in piezoelectric or electrostrictive transducers. The fact, however, that it is magnetic energy stored in the transducer core that is converted, as opposed to electric energy in the other case, requires these two quantities to be differentiated.

The squared coefficient of magnetomechanical coupling in the case of transmitting transducers defines which part (W_p) of the magnetic energy W_m is converted into the mechanical (acoustic) energy W_e. The transformed piezomagnetic energy W_p is the difference in magnetic energy of a transducer between the two mechanical states: at a free state ($T = \text{const}$) and a clamped state ($S = \text{const}$), i.e.

$$W_p = W_T - W_S, \tag{48}$$

$$W_T = \frac{1}{2} HB_T = \frac{1}{2} \mu_T H^2, \tag{49}$$

$$W_S = \frac{1}{2} HB_S = \frac{1}{2} \mu_S H^2, \tag{50}$$

$$k^2 = \frac{W_p}{W_T} = \frac{W_T - W_S}{W_T} = \frac{\mu_T - \mu_S}{\mu_T} = 1 - \frac{\mu_S}{\mu_T}, \tag{51}$$

where H and B are the amplitudes of magnetic field and induction, and μ_T and μ_S are the permeabilities of a free and clamped, respectively.

3.2. *Coefficient of mechanomagnetic coupling*

The opposite case, with respect to the one discussed above, occurs for receiving transducers. In piezomagnetic receivers part of mechanical energy is converted to magnetic energy. The squared coefficient of mechanomagnetic coupling is defined in the following way

$$k^2 = \frac{W_p}{W_H} = \frac{W_H - W_B}{W_H}, \tag{52}$$

where W_H and W_B are the mechanical energies at the constant field intensity H (unloaded, open system) and the constant induction B (closed system), i.e.

$$W_H = \frac{1}{2} S_H T = \frac{1}{2} \frac{T^2}{c_H} = \frac{1}{2} \frac{T^2}{E_H} = \frac{1}{2} s_H T^2, \tag{53}$$

$$W_B = \frac{1}{2} S_B T = \frac{1}{2} \frac{T^2}{c_B} = \frac{1}{2} \frac{T^2}{E_H} = \frac{1}{2} s_B T^2, \tag{54}$$

$$k^2 = \frac{s_H - s_B}{s_H} = \frac{\frac{1}{c_H} - \frac{1}{c_B}}{1/c_B} = \frac{\frac{1}{E_H} - \frac{1}{E_B}}{1/E_H} = 1 - \frac{E_H}{E_B}. \tag{55}$$

4. Relations between the coefficient of magnetomechanical coupling k and the other piezomagnetic coefficients

Relation (46) between the coefficient of magnetomechanical coupling, k, and the piezomagnetic sensitivity d can be derived from the relevant system of piezomagnetic equations I B [5-10, 13]

$$\delta S = \frac{\delta T}{E_H} + d\delta H, \tag{56}$$

$$\delta B = \mu_T \delta H + d\delta T. \tag{57}$$

For a rigid sample (S = const) $\delta S = 0$, and insertion of δT from (56) into (57) gives

$$\delta B_S = \mu_T \delta H - d^2 E_H \delta H. \tag{58}$$

After differentiation with respect to H and division by μ_T (see formula (51))

$$k^2 = \frac{\mu_T - \mu_S}{\mu_T} = \frac{d^2 E_H}{\mu_T}. \tag{59}$$

For a system of equations II B and S = const [7], respectively,

$$\delta S = \frac{\delta T}{E_B} + g\delta B = 0, \tag{60}$$

$$\delta H_S = -g\delta T + \frac{\delta B}{\mu_T} = g^2 E_B \delta B + \frac{\delta B}{\mu_T}. \tag{61}$$

Differentiation with respect to B and transformations lead to a relation connecting the coefficient of coupling, k, with the coefficient of induction strains g:

$$k^2 = \frac{g^2 E_B \mu_T}{1 + g^2 E_B \mu_T}. \tag{62}$$

For the system of piezomagnetic equations III B and a free sample (T = const, $\delta T = 0$) the following relation occurs between the coefficient k and the magnetostrictive constant h:

$$\delta T = E_B \delta S - h\delta B = 0, \tag{63}$$

$$\delta H_T = \frac{\delta B}{\mu_S} - h\delta S = \left(\frac{1}{\mu_S} - \frac{h^2}{E_B}\right)\delta B, \tag{64}$$

$$k^2 = \frac{\mu_T - \mu_S}{\mu_T} = \frac{\mu_S h^2}{E_B}. \tag{65}$$

For the last (IV B) system of piezomagnetic equations [7] in the case of a free sample (T = const) the coefficients k and e are linked by the following relation

$$\delta T = E_H \delta S - e\delta H = 0, \tag{66}$$

$$\delta B_T = e\delta S + \mu_S \delta H = \left(\frac{e^2}{E_H} + \mu_S\right)\delta H, \tag{67}$$

$$k^2 = \frac{\mu_T - \mu_S}{\mu_T} = \frac{e^2}{E_H \mu_T} = \frac{e^2}{e^2 + E_H \mu_S}. \tag{68}$$

Formulae (59), (62), (65) and (68) relate the coefficient of magnetomechanical coupling k with the complete set of 3 coefficients occurring in each of 4 systems of piezomagnetic equations in the SI system. The coefficient k can be defined by 2 (see formulae (51) and (55)) or 3 other piezomagnetic coefficients occurring in different systems of piezomagnetic equations, for example,

$$k^2 = 1 - \frac{\mu_S}{\mu_T} = 1 - \frac{E_H}{E_B} = eg = \frac{dh}{1 + dh}, \tag{69}$$

$$k^2 = \mu_S gh = dgE_H = \mu_S g^2 E_B = \mu_T g^2 E_H, \tag{70}$$

$$k^2 = \frac{eh}{E_B} = \frac{ed}{\mu_T} = \frac{e^2}{\mu_T E_H} = \frac{e^2}{\mu_S E_B}. \tag{71}$$

5. Relations between the coefficients of magnetomechanical coupling defined from equations of type B and M

In piezomagnetic equations in which the variable magnetic quantities are the field intensity H and the magnetization M (or the magnetic polarisation I or J), i.e. in the M type equations, e.g. [1, 2, 6, 7, 10, 12, 15, 16], the coupling coefficient is defined as

$$k^2(M) = 1 - \frac{\varkappa_S}{\varkappa_T} = 1 - \frac{E_H}{E_M}. \tag{72}$$

Division of equation (51) by the first part of equation (72) gives the following ratio

$$\frac{k^2(B)}{k^2(M)} = \frac{(\mu_T - \mu_S)\varkappa_T}{\mu_T(\varkappa_T - \varkappa_S)} = \frac{(\varkappa_T - \varkappa_S)\varkappa_T}{(\varkappa_T + \mu_0)(\varkappa_T - \varkappa_S)} = \frac{\varkappa_T}{\varkappa_T + \mu_0}$$

$$= \frac{\varkappa_T}{\mu_T} = \frac{1}{1 + \mu_0/\varkappa_T} = 1 - \frac{\mu_0}{\varkappa_T}. \tag{73}$$

In magnetic materials of permeability from 10 to 100

$$k^2(B) = (0.9-0.99)k^2(M), \tag{74}$$

i.e. differences between k may be less than 0.5 per cent and below 5 per cent.

Consideration of the final terms of relations (55) and (72) gives

$$\frac{k^2(B)}{k^2(M)} = \frac{1-E_H/E_B}{1-E_H/E_M} = \frac{(E_B-E_H)E_M}{E_B(E_M-E_H)}. \tag{75}$$

6. Definitions of the coefficient of magnetomechanical coupling in the nonrationalized cgsm unit system

In piezomagnetic equations in the nonrationalized magnetic cgs system of type B_c, the coefficient 4π occurs in one equation of each system, and the formulae relating the coupling coefficient k with the other piezomagnetic coefficients have the following form [6-12]

$$k^2 = \frac{d^2E_h}{4\pi\mu_T} = \frac{\Lambda^2 E_H}{4\pi\mu_T} = \frac{4\pi\mu_T g^2 E_B}{1+4\pi\mu_T g^2 E_B} = \frac{4\pi\mu_S h^2}{E_B}$$

$$= \frac{4\pi\mu_S\lambda^2}{E_B} = \frac{e^2}{e^2+4\pi\mu_S E_H} = 1-\frac{E_H}{E_B} = 1-\frac{\mu_S}{\mu_T}, \tag{76}$$

while the relation between the systems of type B and those of type M_0 is the following [6, 7]

$$k^2(B) = \frac{k^2(M_c)}{1+\mu_0/4\pi\mu_T}. \tag{77}$$

7. Conclusion

Most of piezomagnetic materials used so far are polycrystals for which the coefficient of magnetomechanical coupling k is defined for longitudinal and radial vibration, i.e. k_{33}, or for torsional vibration, k_{51}, e.g. [1, 2]. Coupling in a material or a piezomagnetic transducer is defined by the characteristic $k = f(H)$ for the primary magnetization curve and the hysteresis loop (Figs. 1, 2).

In the state of demagnetization and magnetic saturation, the magnetomechanical coupling vanishes ($E_H = E_B$, $\mu_T = \mu_S$, $k = 0$), and definition of characteristics of the coupling coefficient involves indirect determination of the state of magnetization of a material. The maximum values of k and the values of k in the remanence state are given in a short form (k_m, k_r). In good piezomagnetic materials $k > 0.15$ and, for example, in the metglasses of the Fe–Si–B alloys and in the rare earth alloys, k reaches now 0.85.

Coupling measurements show that saturation of magnetic material requires much larger fields than those in the standards of many renowned companies, which take, for example, 2,4 or 4 kA/m (30 or 50 Oe). The characteristics of k are also determined as a function of temperature, e.g. [1, 2, 4, 8, 9, 11, 14]. Synthetic

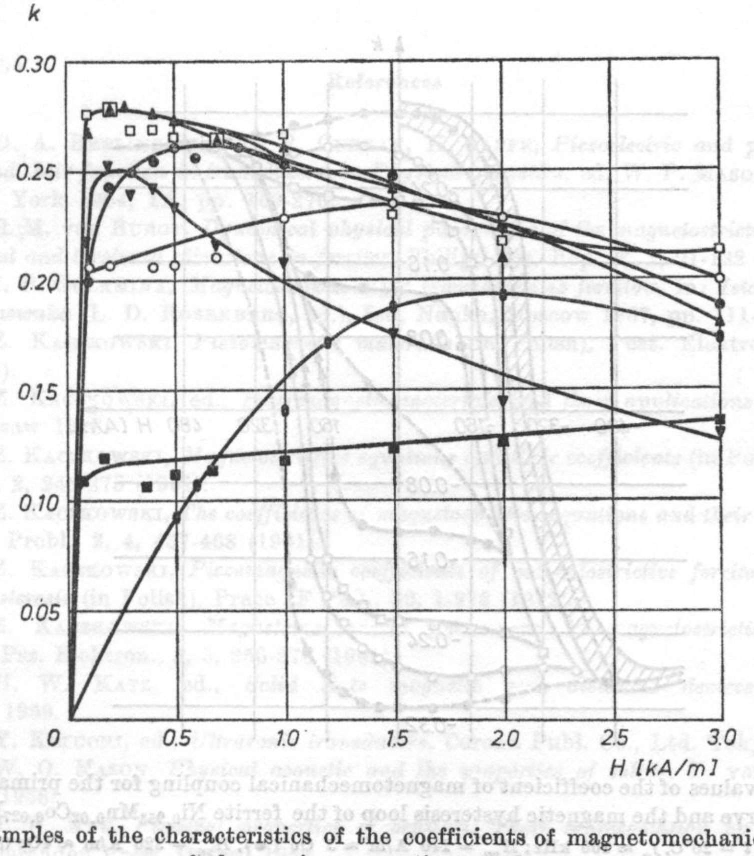

Fig. 1. Examples of the characteristics of the coefficients of magnetomechanical coupling of closen piezomagnetic transducers

n – unjoined transducer, l – soldered transducer, k – cemented transducer, alfer: (▲) k, (□) l; nickel: (o) n, (■) l; permendur (●) n, alcofer (▼) l; ferrite $Ni_{0.968}Co_{0.012}Mn_{0.02}Fe_2O_4$(●); $f_r \approx$ 22-28 kHz

definitions and relations permit the comparison of the results obtained by authors using different definitions and systems of equations and units; or on the basis of knowledge of the values of the coefficient of magnetomechanical coupling, and moduli of elasticity and permeability, the determination of the other piezomagnetic coefficients occurring in piezomagnetic equations.

With a sufficient quality factor, the coefficient of magnetomechanical coupling can be defined from the mechanical resonance frequency f_r and the magnetomechanical antiresonance f_a, or from the ultrasonic velocities, proportional to these, for constant field intensity c_H or for constant induction c_B from the following relation

$$k = \sqrt{1-\left(\frac{c_H}{c_B}\right)^2} \approx \sqrt{1-\left(\frac{f_r}{f_a}\right)^2} \approx \sqrt{2\left(1-\frac{f_r}{f_a}\right)}. \tag{78}$$

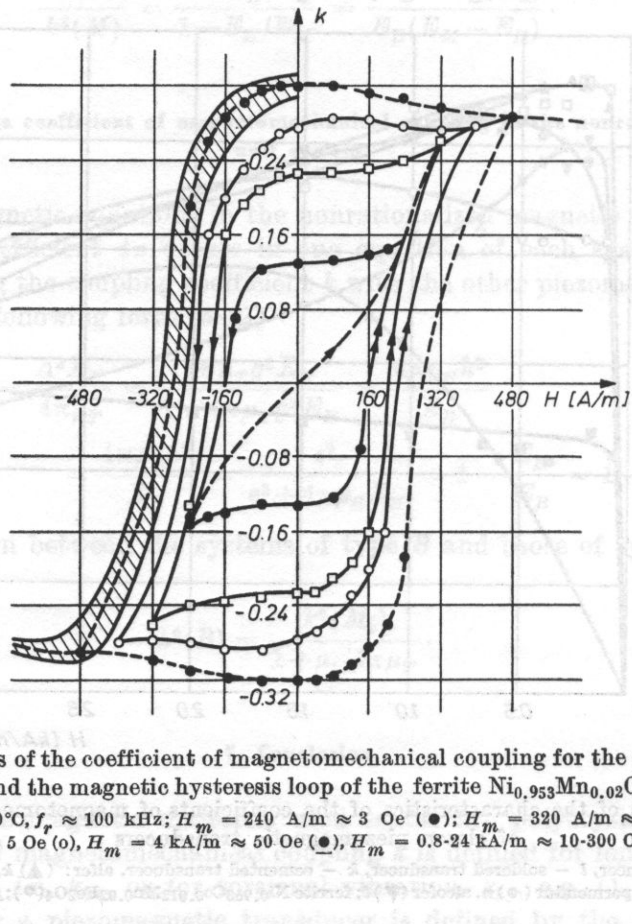

Fig. 2. The values of the coefficient of magnetomechanical coupling for the primary magnetization curve and the magnetic hysteresis loop of the ferrite $Ni_{0.953}Mn_{0.02}Co_{0.027}Fe_2O_4$ [8]

H = 0.2 A/m, t = 20°C, $f_r \approx$ 100 kHz; H_m = 240 A/m ≈ 3 Oe (●); H_m = 320 A/m ≈ 4Oe (□); H_m = 400 A/m ≈ 5 Oe (o), H_m = 4 kA/m ≈ 50 Oe (●̲), H_m = 0.8-24 kA/m ≈ 10-300 Oe (- - -)

The validity of the proportional relation between the resonance frequencies and the velocities depends on their high mechanical quality factor Q_m of such a value that the product $k^2\ Q_m \geqslant 10$. In addition to the mechanical quality factor Q_m and the magnetic quality factor, the coefficient k is one of the basic parameters defining the efficiency of a material or transducer. From these quantities the so called electromechanical efficiency η_{em} can, for example, be defined [1-3, 10-12, 14],

$$\eta_{em} = \frac{k^2 Q_m Q_\mu}{1+k^2 Q_m Q_\mu}. \tag{79}$$

Thus, the materials with a high k are not always optimum, since their quality factor can be very low and, as a result, their efficiency will be worse, e.g. in ferrites and alfers.

The knowledge of the coupling coefficient and the quality factor permits piezomagnetic and piezoelectrical to be compared.

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