Archives of Acoustics Vol. **51**, No. 2, pp. 235–247 (2026), https://doi.org/10.24423/archacoust.2026.4408

Research Paper

Construction Method of Sparse Dictionary for Multi-Order FRFT Domain Feature Fusion

Tongjing Sun*, Lei Chen, Xiaohong Deng

Department of Automation, Hangzhou Dianzi University
Hangzhou, China

*Corresponding Author: stj@hdu.edu.cn

Received January 5, 2026; accepted March 2, 2026; available online March 10, 2026; version of record April 24, 2026; published issue June 24, 2026.

Reverberation constitutes a primary source of interference for active sonar signals, particularly the intense reverberation originating from reflections of the incident signal. Sharing the same generation mechanism as the target echo, it severely hampers the extraction and analysis of the target signal. To enhance signal processing capabilities under strong reverberation, this paper proposes a sparse dictionary construction method based on multi-order fractional Fourier transform (FRFT) domain feature fusion. This method exploits the distinctive characteristics exhibited by target echoes and strong reverberation signals across different fractional transform domains to discriminate between them. It constructs sparse sub-dictionaries using these distinct fractional orders, trains the weights of each sub-dictionary via an adaptive gradient optimization strategy to achieve sparse representation of the signal, suppresses strong interference in the sparse domain, and reconstructs the target signal through a reconstruction process, thereby achieving the goal of extracting the target signal while suppressing strong interference. Results from processing lake trial data demonstrate that the proposed method can effectively extract target echo signals amidst strong reverberation, with the signal-to-reverberation ratio improvement consistently no less than 2.1 dB and reaching up to 15.6 dB. This method provides an effective approach for the processing and analysis of weak underwater signals.

Keywords: underwater acoustic signal processing, fractional Fourier transform, feature fusion, sparse reconstruction, strong interference suppression.

1. Introduction

With the continuous advancement of marine exploration technology, active sonar systems play an increasingly vital role in underwater target detection, recognition, and localization (Cui *et al.*, 2023). However, the actual marine environment is complex and variable, containing various noise sources such as waves, wind-generated surface noise, and marine life, all of which interfere with the analysis of target echoes in sonar signals (Yang, 2023). Among these, reverberation stands as a primary background interference for active sonar, especially strong reverberation induced by the transmitted signal. Its spectral structure exhibits a certain similarity to the transmitted signal, rendering conventional time-domain filtering, frequency-domain suppression, and matched filtering methods inadequate for reverberation cancellation (Deng *et al.*, 2005). Consequently, under conditions of strong reverberation interference, traditional methods struggle to effectively extract target echoes, which constitutes a key bottleneck limiting performance enhancement in sonar systems.

Sparse representation theory, which describes signals succinctly using a small number of atoms from an over-complete dictionary, can better reveal, distinguish, and extract the informational features inherent in signals. This offers a new perspective and methodology for extracting underwater target signals (Ge, Zhang, 2023).

In the past, scholars have conducted related research in this area. Lei (2014) leveraged the structural characteristics of multi-layer wavelet decomposition in conjunction with wavelet modulus maxima line search theory to achieve sparse signal representation based on wavelet modulus maxima search. Sun *et al.* (2016), addressing the issue of processing weak underwater echo signals, proposed a method for constructing an overcomplete atomic library based on prior information, building a library tailored to the signal's own characteristics to achieve sparse reconstruction of underwater echo signals. Liu and Zhou (2018) constructed a joint dictionary for time-frequency overlapping communication signals to perform sparse representation and interference suppression. Sparse representation methods are also widely applied in tasks such as underwater acoustic signal denoising, signal direction finding, and target classification (Zhou *et al.*, 2023; Wu *et al.*, 2021; Meng, 2024; Liao *et al.*, 2014; Wang, 2020). These methods typically construct dictionaries using wavelet bases or the incident signal itself. The principle behind interference suppression relies on the correlation between the echo signal and the incident wave, while noise or other interferences are uncorrelated with it. This allows environmental noise or random scatterers to be filtered out. However, for strong reverberation that shares the same generation mechanism as the target echo signal, it becomes difficult to achieve target signal extraction. To suppress strong reverberation, the constructed overcomplete dictionary must resemble the features of the target signal as closely as possible while differing from those of the strong reverberation signal. This enables the elimination of the reverberation component during the sparse representation process, leaving only the target signal and thereby achieving both target extraction and reverberation suppression.

For separating target signals from reverberation interference, the fractional Fourier transform (FRFT) has gained attention because it can achieve energy concentration on chirp basis functions, potentially aiding in reverberation suppression. Therefore, many researchers employing FRFT for processing signals in reverberant backgrounds. Yu and Park (2017) used FRFT to obtain Doppler shift, range, and azimuth information of echo signals in strong ocean reverberation. Zhang *et al.* (2014) introduced FRFT into echo signal detection, using a sliding window to segment the echo signal and extracting the start position of the echo when the segmented signal's peak location in the FRFT domain matched that of the transmitted signal. Liang *et al.* (2024) proposed a frequency-domain adaptive matched filter detection method based on FRFT, achieving target detection in reverberation backgrounds through optimal-order transformation and template matching. However, these studies primarily focus on where reverberation energy does not concentrate on the optimal FRFT order. When the reverberation signal shares the same generation mechanism as the target signal, its energy may also concentrate at the optimal order, making it impossible to suppress the reverberation signal using only optimal-order information.

Therefore, considering that FRFT not only effective target features at the optimal order but also represents features on different time-frequency planes at various fractional orders, its multi-order characteristics serve as a powerful basis for distinguishing sonar targets and can be utilized to differentiate between reverberation and target echo signals. Consequently, our study analyzes the energy distribution characteristics of target echoes and reverberation across multiple fractional orders and selects feature sub-domains with strong discriminative power. Subsequently, sparse sub-dictionaries are constructed within each selected sub-domain. An adaptive gradient optimization strategy (Fan, 2024) is employed to train the fusion weights for these multiple domains, achieving feature fusion over multiple fractional orders. The final fused dictionary significantly enhances target focusing and interference suppression capabilities in strong reverberation backgrounds, thereby improving the extraction capability of active sonar target echoes.

2. Sparse representation theory

Sparse representation theory involves representing or approximating a signal using a linear combination of as few atoms as possible from an overcomplete dictionary. Under sparse representation, most of the signal's energy is concentrated in a small number of significant positions, while noise exists in a more dispersed form. Therefore, by extracting a limited number of energy-concentrated feature points, it is possible not only to retain the primary information of the target but also to suppress background noise.

The sparse representation problem is typically modeled as a non-convex optimization problem:

$$\min_{x} \|x\|_0 \qquad \text{subject to} \qquad y = Dx, \tag{1}$$

where $D = \{g_\gamma\}_{\gamma \in \Gamma}$ is an overcomplete dictionary whose elements g_γ are called atoms, and the number of atoms far exceeds the signal dimension N, and $\|x\|_0$ denotes the l_0 – norm, representing sparsity – the number of non-zero elements in the sparse coefficient vector. The objective is to represent the original signal y using as few atoms as possible. Figure 1 illustrates the principle of sparse representation.

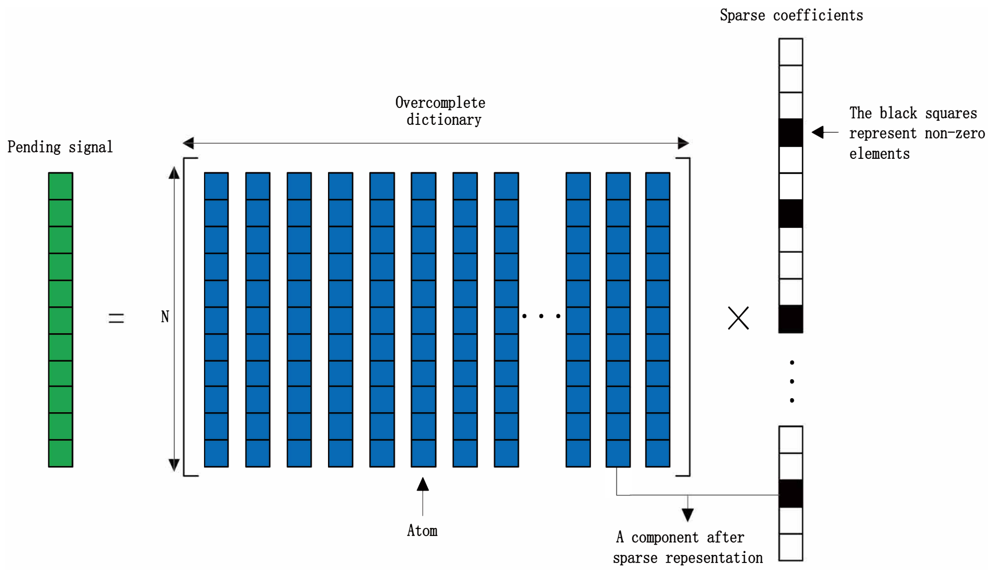

Fig. 1. Schematic diagram of the sparse representation principle.

Sparse representation theory primarily consists of two components: the overcomplete dictionary and the sparse decomposition algorithm. Regarding the construction of the overcomplete dictionary, traditional methods are generally based on time-frequency analysis and are suitable for signals exhibiting sharp local feature variations in the time-frequency domain. If the goal is to represent such signals more sparsely, the corresponding dictionary must encompass richer time-frequency features, thereby introducing redundancy compared to the original time-frequency transform dictionary (Liu *et al.*, 2008).

Let us assume $g(t) = L^2(R)$ is a general window function that satisfies four conditions: (1) it is a continuously differentiable real function, (2) $g(t) \in O\left(1/(t^2+1)\right)$, (3) its norm is 1, i.e., it is equivalent to $\|g\| = 1$, and (4) its integral over the entire real number line is non-zero, i.e., $g(0) = 0$. By applying scaling transformations, time shifts, and modulation to $g(t)$, a generalized overcomplete representation dictionary can be obtained. The characteristics of a well-structured overcomplete dictionary are as follows: first, the dictionary contains a sufficient number of atoms with a wide variety to achieve sparse signal representation and yield high-quality decomposition results, and second, the atoms within the overcomplete dictionary should exhibit significant differences, ensuring storage and computational efficiency. As for the sparse decomposition algorithm, its core lies in iteratively approximating the signal, during which components mismatched with the target structure are eliminated and effective information is retained. For instance, the matching pursuit algorithm selects, in each iteration, the atom from the overcomplete dictionary that exhibits the strongest correlation with the signal to be decomposed.

3. Construction method of the fusion dictionary in the multi-order FRFT domain

The overall workflow of the proposed method is illustrated in Fig. 2, which embodies the core concept of ‘multi-order feature extraction + weighted fusion + sparse reconstruction.’

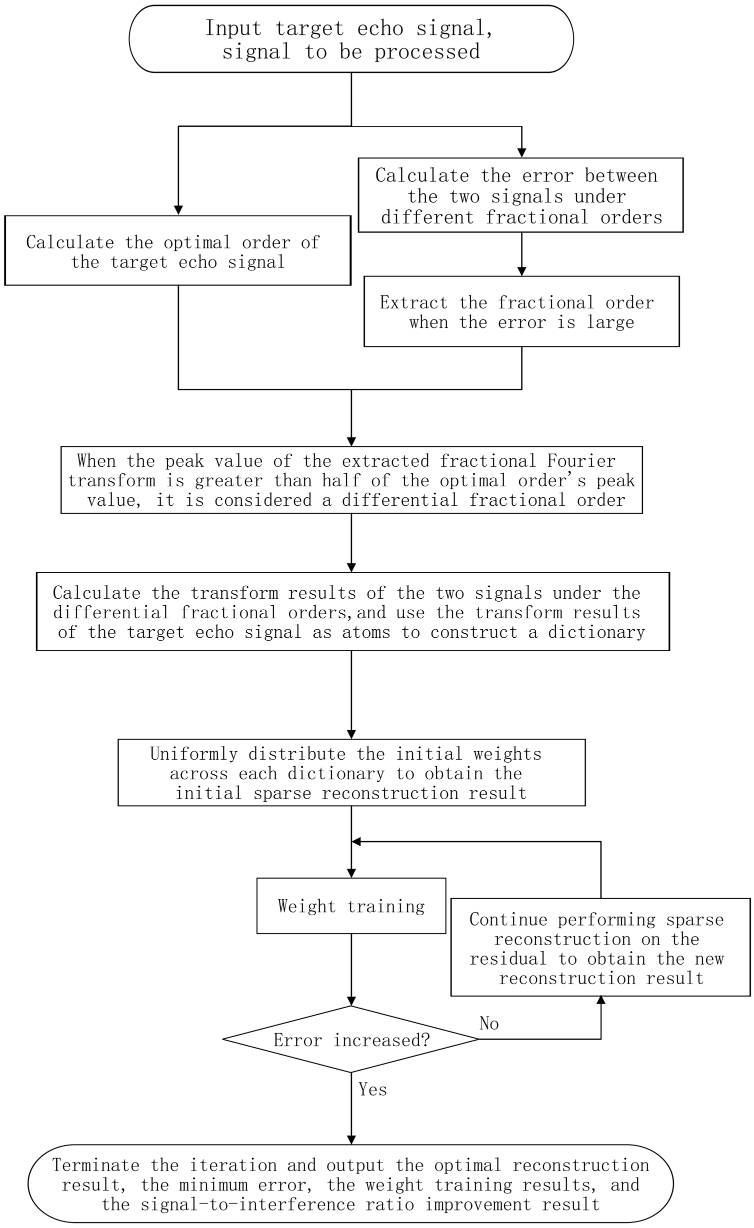

Fig. 2. Flowchart of sparse dictionary construction based on multi-order FRFT domain feature fusion.

The FRFT is a generalized form of the conventional Fourier transform. By introducing a fractional order parameter, it defines a transform domain situated between the time and frequency domains (Ma *et al.*, 2018). Compared to the conventional Fourier transform, which provides only frequency-domain information, the FRFT reveals different features across various time-frequency planes. It offers significant advantages when processing echo signals containing linear frequency modulation (chirp) characteristics. Multi-order FRFT features can capture the distinguishing characteristics between the target signal and strong reverberation (Weng, 2020).

Therefore, the fundamental idea of this paper is as follows: by extracting the differential features between the target echo signal and strong reverberation across different fractional orders, a multi-order sparse dictionary is constructed. Subsequently, an adaptive weighted fusion mechanism is employed to optimize the feature components within each dictionary that contribute most significantly to the target, thereby suppressing the interference signal. Finally, target signal separation and enhancement are achieved through sparse reconstruction.

This method primarily consists of two parts: differential fractional order extraction and fusion dictionary weight training, which are elaborated in Subsec. 3.1 and Subsec. 3.2, respectively.

3.1. Differential fractional order extraction method

In the actual processing of echo signals, since the generation mechanism of strong reverberation is similar to that of the target echo signal, the objective is to identify fractional orders in the FRFT domain that capture the differences between the strong reverberation and the target echo signal. These selected orders are then used to construct a multi-domain dictionary for separating the target echo from the strong reverberation. The specific steps are as follows:

1. Optimal fractional order estimation:
   Apply the FRFT method based on the maximum peak criterion to perform FRFT on the target echo signal. Identify the fractional order at which the signal's energy is most concentrated in the transform domain, denoted as the optimal order. The amplitude at this order serves as the baseline reference for subsequent differential order extraction.
2. Error difference calculation:
   Perform the FRFT on both the target signal and the strong reverberation signal separately across a predefined set of fractional orders. Calculate the error (using the Euclidean distance method) between them at each order to evaluate their structural differences under different fractional orders.
3. Differential fractional order selection:
   Select those fractional orders where the error in the FRFT domain is relatively large and the signals exhibit distinct energy concentration as the differential fractional orders. The selection criterion is that the peak energy at a candidate order is not lower than a specified proportion of the peak energy at the optimal fractional order. These differential orders are considered structurally more suitable for distinguishing the target signal from the interference signal.

Through the such steps, a set of differential fractional orders is obtained. These orders are used to construct the multi-domain sparse dictionary, with each fractional order corresponding to a sub-dictionary within this domain, forming the initial framework for multi-feature fusion sparse representation.

3.2. Weight training method for the fusion dictionary

Suppose P differential fractional orders are extracted. Each order corresponds to a sparse sub-dictionary D_i and is assigned a weight w_i. The construction of each dictionary is based on the FRFT results at that specific order as the atom set, forming dictionaries containing prior feature information. To achieve effective fusion of information from multiple fractional orders, this research designs a weight training method based on gradient descent. By incorporating historical gradient information and neighborhood fluctuation estimation, the learning rate is adaptively adjusted to optimize the contribution of each dictionary to the overall sparse representation.

The specific process is as follows:

1. Weighted model for sparse reconstruction results:
   For the sparse dictionary set constructed from P differential fractional order domains, the overall sparse reconstruction result can be expressed as a weighted superposition of the reconstruction results from each sub-dictionary:
   $$S_{re} = \sum_{i=1}^{P} w_i \varphi_i = \sum_{i=1}^{P} w_i D_i \alpha_i, \tag{2}$$
   where S_{re} is the weighted reconstruction result, φ_i is the reconstruction result corresponding to the i-th dictionary, D_i is the sub-dictionary constructed from the i-th differential fractional order, α_i is the sparse coefficient under that dictionary, and w_i is the corresponding fusion weight. The goal is to train the weights w_i so that the fusion result approximates the original target echo signal S as closely as possible, thereby achieving more accurate reconstruction of the target information.
2. Loss function and gradient calculation:
   To this end, the gradient descent method is introduced for weight training. Aiming to minimize the reconstruction error, the loss function is defined as

$$L = \|S - S_{re}\|_2^2 = \left\|S - \sum_i^P w_i\varphi_i\right\|_2^2 = \|S - \phi w\|_2^2 = (S - \phi w)^T (S - \phi w) = S^T S - S\phi w - w^T\phi^T S + w^T\phi^T\phi w, \quad (3)$$

where S is the target echo signal, $\phi = [\varphi_1, \varphi_2, ..., \varphi_P]$, and $w = [w_1, w_2, ..., w_P]^T$. The gradient corresponding to weight w_i is given by

$$\mathrm{Grad}_i = \partial L/\partial w_i = \sum_{j=1}^P w_j\varphi_i^T\varphi_j + \sum_{j=1}^P w_j\varphi_j^T\varphi_i - S^T\varphi_i - \varphi_i^T S, \qquad i = 1, 2, ..., P. \quad (4)$$

3. Curvature estimation and historical information recording:
To further improve the convergence efficiency of weight updates and enhance the stability of the training process, the second-order derivative of w_i is introduced as $L^{(2)} = 2\varphi_i^T\varphi_i$, which aids in evaluating the curvature of the objective function and informs step size selection. Simultaneously, considering that gradients may fluctuate sharply during actual training, a history-based approach is used to dynamically adjust the learning rate, and update the weights based on past gradient and loss information. The historical maximum absolute gradient is denoted as

$$\mathrm{MaxGrad} = \max\left(\partial L/\partial w_i^{(t)}\right), \quad (5)$$

and the historical maximum absolute loss function value is

$$\mathrm{MaxLoss} = \max\left(\left|L\left(w^{(t)}\right)\right|\right). \quad (6)$$

4. Adaptive neighborhood range adjustment:
To avoid local optima or convergence stagnation due to an excessively small learning rate, an adaptive neighborhood range (Range) is defined, which is gradually adjusted during the iteration process. When approaching the optimum, the local search range for learning samples should be appropriately expanded. The variable Range is obtained as follows:

$$\mathrm{Range} = \log_2(1 + \log_{R_b} R_r), \quad (7)$$

where

$$R_b = 1 + |L'|/\mathrm{MaxGrad} = 1 + \left|\mathrm{Grad}_i'\right|/\mathrm{MaxGrad}, \quad (8)$$

$$R_r = 1 + (|L| + \mathrm{MaxLoss})/\mathrm{MaxLoss}. \quad (9)$$

5. Weight update strategy:
Based on the aforementioned calculations, the main weight update rule is

$$\begin{aligned} w_i &= w_i + LR \times L' = w_i + \frac{\log_{(2+\mathrm{FlucDegree})}(1 + |\mathrm{Grad}_i|)}{|\mathrm{Grad}_i| + \mathrm{FlucDegree}} \times \mathrm{Grad}_i \\ &= w_i + \frac{\log_{(2+\mathrm{Cur}+\mathrm{AdaVar})}(1 + |\mathrm{Grad}_i|)}{|\mathrm{Grad}_i| + \mathrm{FlucDegree}} \times \mathrm{Grad}_i, \end{aligned} \quad (10)$$

where FlucDegree reflects the fluctuation degree of the first-order derivative, and Cur is the current weight. The corresponding computational formulas are:

$$\mathrm{Cur} = \frac{\left|L^{(2)}\right|}{(1 + L'^2)^{3/2}}, \quad (11)$$

$$\mathrm{FlucDegree} = \log_{(2\times\mathrm{Range})}(1 + \mathrm{Var}(\varphi_i)). \quad (12)$$

6. Overall training process:

   a) Initialize weights and sub-dictionaries: initialize weights w_i^0 for all sub-dictionaries generated from differential fractional orders. Perform the first sparse reconstruction to obtain the initial reconstruction result and initial dictionary coefficients.

   b) Alternating update iteration:

      ① fix the sparse coefficients α_i, update the weights w_i,

      ② after updating the weights, perform sparse reconstruction again to obtain new coefficients α_i,

      ③ repeat the prior process until the reconstruction error converges or the preset number of iterations is reached.

   c) Output final results: output the optimal sparse reconstruction result S_{re} and the minimized reconstruction error, which will be used for subsequent signal reconstruction or interference suppression.

Through the aforementioned optimization method, the fusion sparse dictionary, constructed in this study, can adaptively capture the energy characteristics of the target echo signal across different fractional orders, effectively enhancing signal reconstruction quality and interference suppression capability.

4. Experimental results and analysis

4.1. Experimental overview

The data processed in this paper originates from lake trial experiments conducted in the Xin'anjiang waters of Jiande City, Hangzhou, Zhejiang Province. The experimental target was a spherical model. The experimental deployment is shown in Fig. 3. During the experiment, the target moved from far to near or from near to far. A total of 150 test signals were selected for processing. The experiment employed a monostatic active testing system transmitting a linear frequency modulated (LFM) signal with the following parameters: frequency sweep bandwidth of 40 kHz, center frequency of 60 kHz, pulse width of 5 ms, and a sampling frequency of 1 MHz.

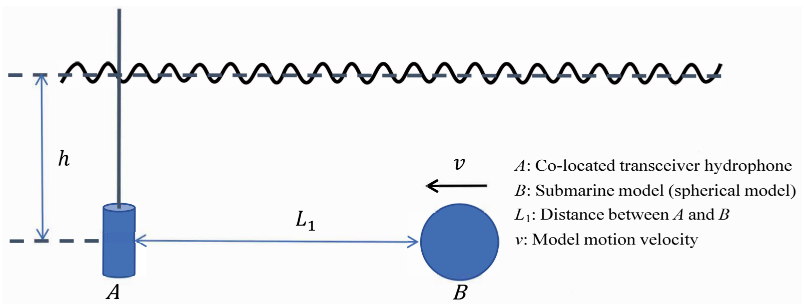

Fig. 3. Experimental deployment.

4.2. Analysis of measured data processing

Based on the test results selected in the previous subsection, where the target's motion was from far to near, the acquired signals were used to construct signals to be processed under different signal-to-reverberation ratios (SRR). The waveform of the 75th test signal under different SRR conditions (3 dB and 5 dB) is shown in Fig. 4.

First, the signal with an SRR of 3 dB was selected as the signal to be processed. To compare the sparse representation performance between the FRFT domain and the time domain, dictionaries were constructed using time-domain and FRFT domain features, respectively. The resulting sparse coefficient plots are shown in Fig. 5.

Based on the sparse coefficient plots, it can be observed that the sparse coefficients for the time-domain dictionary are relatively dispersed, while those for the FRFT domain dictionary are more concentrated. This indicates that the FRFT domain dictionary can better separate the target signal from the reverberation background.

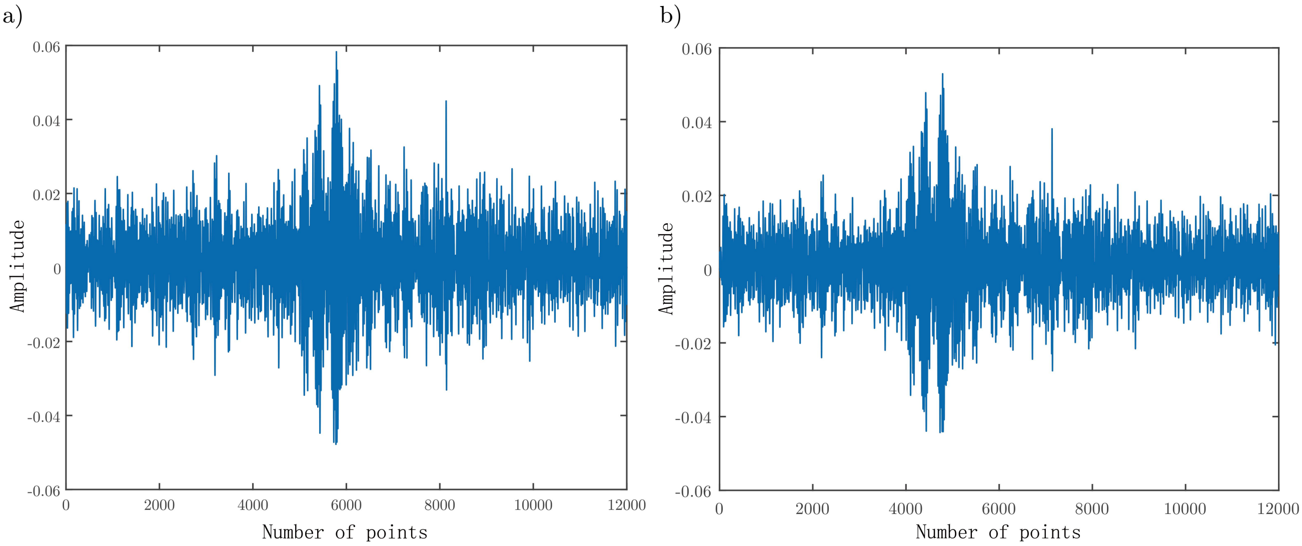

Fig. 4. Waveform of the 75th test signal under different SRR conditions: a) SRR is 3 dB, b) SRR is 5 dB.

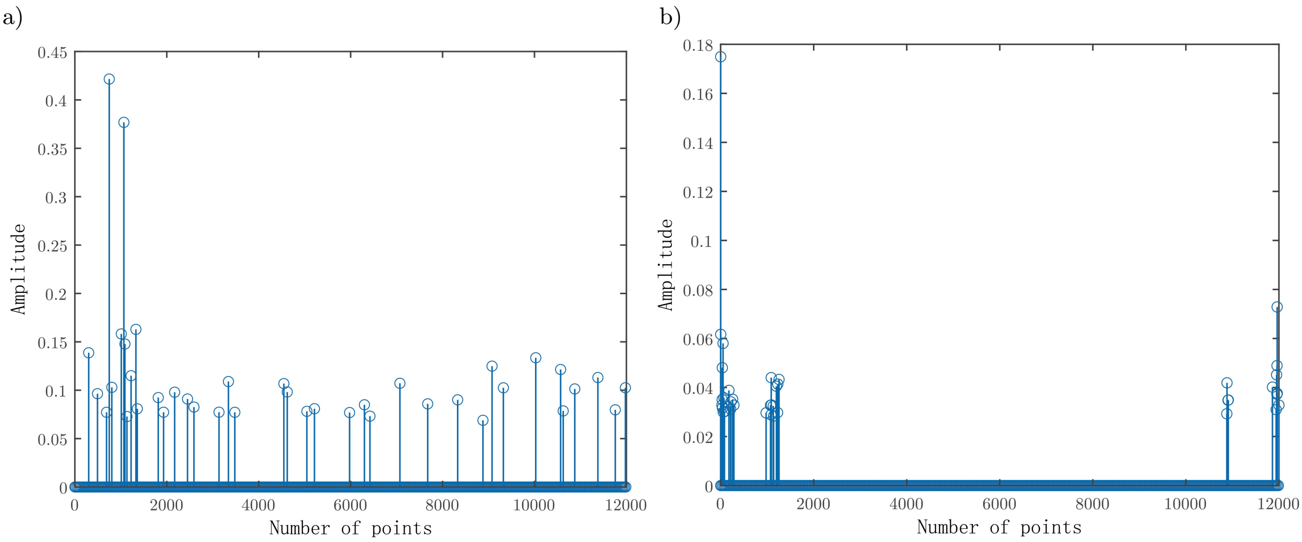

Fig. 5. Comparison of sparse coefficients in time domain vs. FRFT domain:
a) time-domain sparse coefficient plot, b) FRFT domain sparse coefficient plot.

To extract the target echo signal under strong reverberation, the last measurement signal was used as the target signal, and the corresponding reverberation portion from that measurement was extracted to derive the differential fractional orders. Figure 6a shows the amplitude-normalized waveform of the target signal, and Fig. 6b shows the amplitude-normalized waveform of the reverberation signal.

Following the workflow illustrated in Fig. 2, the FRFT was first applied to the target signal to find its optimal fractional order. Using the maximum peak criterion, it was determined that the signal exhibits the strongest energy concentration at a specific fractional order, which is designated as the optimal order for the target signal. The FRFT result of the target echo signal at this optimal order is shown in Fig. 7a, and the corresponding transformation result of the reverberation signal at the same order is shown in Fig. 7b.

When comparing the results of the target echo and reverberation signals at the optimal fractional order in Fig. 7, it can be seen that the reverberation signal also exhibits energy concentration at the target's optimal order, although its concentration amplitude is significantly lower than that of the target signal. Nevertheless, the concentration degree in the reverberation signal is still relatively high. Therefore, it is necessary to identify more discriminative fractional orders for feature-domain processing. According to the proposed method, the differences

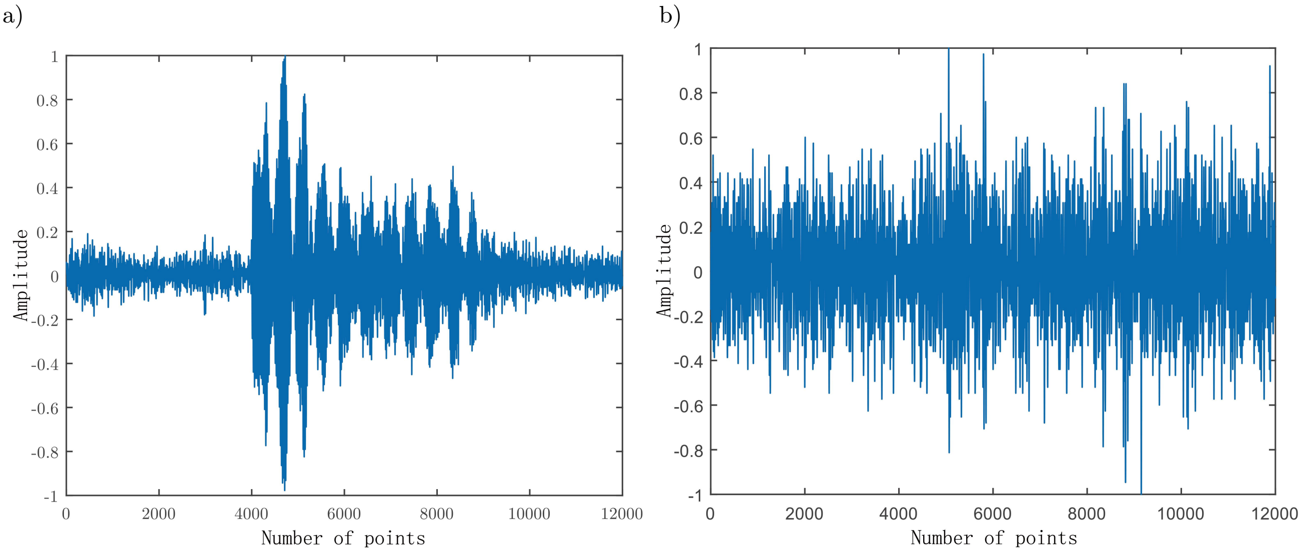

Fig. 6. Amplitude-normalized waveforms of target echo signal (a) and reverberation signal (b).

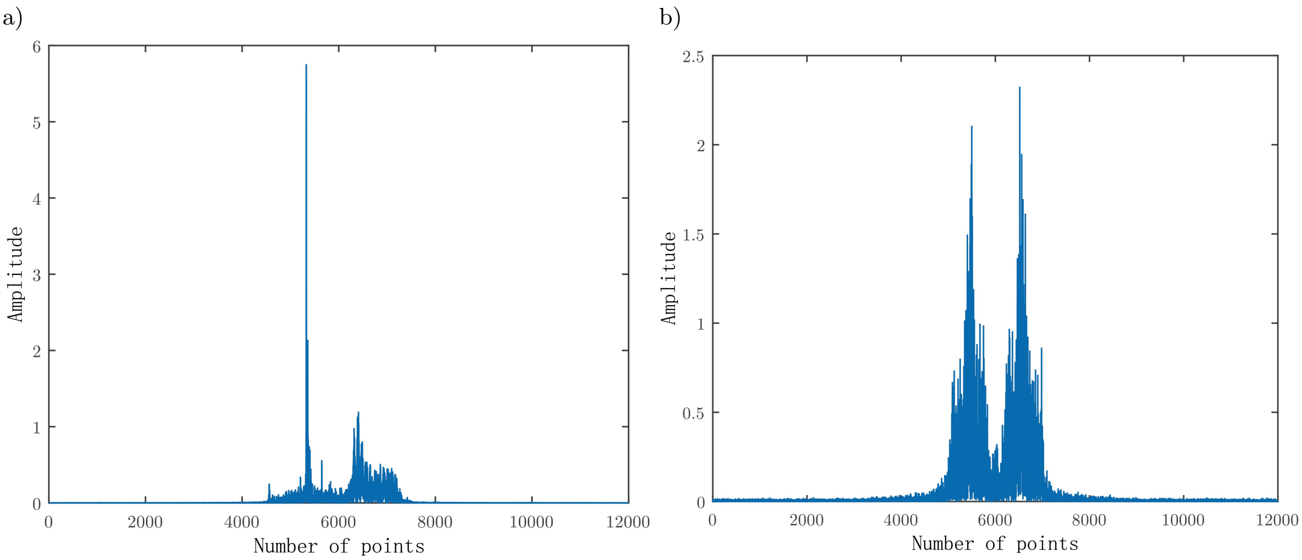

Fig. 7. FRFT results at the optimal fractional order of target signal (a) and reverberation signal (b).

between the target and reverberation signals across different fractional orders were calculated to determine the final set of differential fractional orders, which serve as the basis for constructing the sparse representation dictionary. The calculated differential fractional orders for the target and reverberation signals are:

$$P_{\text{diff}} = [0.939,\ 0.938,\ 0.940,\ 0.937,\ 0.936]. \tag{13}$$

Based on the obtained differential fractional orders, the proposed method was used to construct sparse representation dictionaries in each corresponding fractional order domain. The signal to be processed was then analyzed. The final trained weight distribution was $[w_1, w_2, w_3, w_4, w_5]$ = [0.1642, 0.1569, 0.1801, 0.2324, 0.2664]. The reconstruction result is shown in Fig. 8a, with a corresponding minimum reconstruction error of $\xi_{\min} = 0.68561$. The reconstructed SRR was 18.06 dB, representing an improvement of 15.06 dB. Simultaneously, the proposed method was applied to process the 75th test signal with an initial SRR of 5 dB, and the processing result is shown in Fig. 8b. By comparing the signals before and after reconstruction, it is evident that the target signal, originally submerged in reverberation, has been effectively extracted.

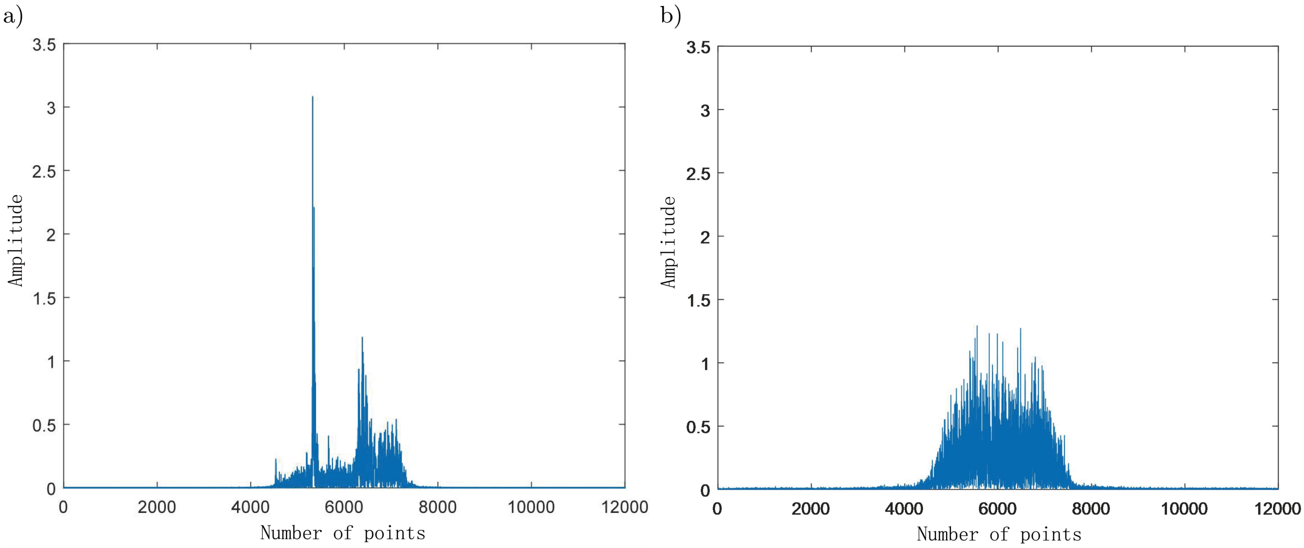

Fig. 8. Sparse reconstruction results (of the 75th test signal) under different initial SRR conditions for SRR = 3 dB (a) and SRR = 5 dB (b).

Figure 9 shows the SRR improvement for the 75th test signal across different initial SRR conditions. It demonstrates that the proposed method maintains high target signal extraction performance even in strong reverberation backgrounds.

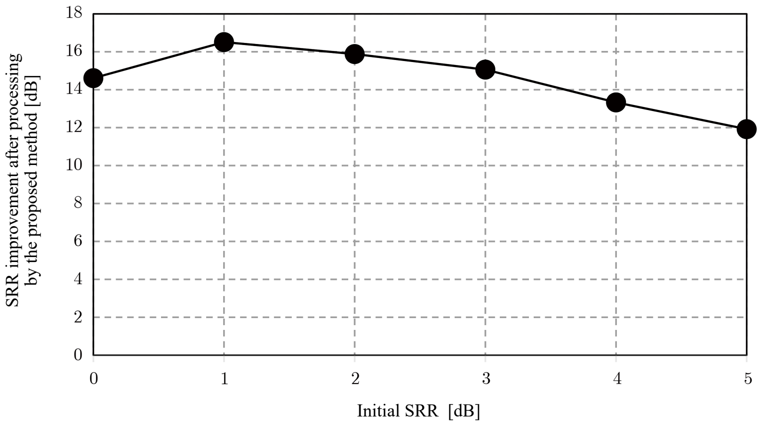

Fig. 9. SRR improvement under different initial SRR conditions.

The SRR improvement for different test signals under various initial SRR conditions were obtained, and the data are presented in Table 1.

Table 1. SRR improvement [dB] for different test signals under different initial SRR conditions.

Test signals	Initial SRR					
	0	1	2	3	4	5
25th	6.53	12.17	10.97	9.74	9.08	8.15
55th	13.96	13.88	12.67	11.27	10.24	9.18
110th	10.48	9.58	8.71	7.67	10.28	9.27
140th	7.46	6.52	12.46	11.60	10.64	9.77

Based on the data in Table 1, it can be concluded that the post-processing SRR improvement is influenced not only by the initial SRR but also by the target's position. When the target is far away and interference is strong, extracting the target signal is more difficult. Furthermore, since measured reverberation was used to construct signals with different initial SRR levels, the actual SRR is lower than the values calculated in this paper, which also affects the observed improvement. However, the data show that the proposed method achieves effective improvement within the selected SRR range (0 dB to 5 dB), with an improvement of no less than 5 dB, confirming the robustness of the method.

To further validate the method's performance across all test signals, processing was conducted on all test signals under SRR conditions of 3 dB and 5 dB. The SRR improvement for different test signals under these conditions is shown in Fig. 10.

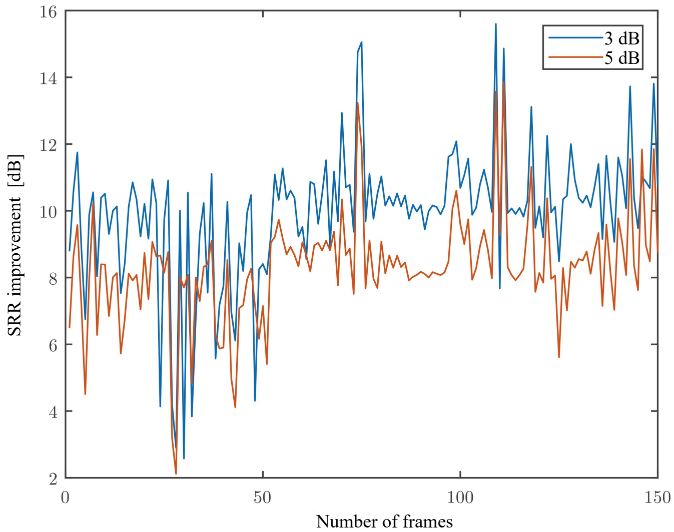

Fig. 10. SRR improvement across all test signals under initial SRR conditions of 3 dB and 5 dB.

Based on the aforementioned processing results, it is concluded that the proposed method successfully implements sparse dictionary construction based on multi-order FRFT domain feature fusion. Experimental results verify the feasibility of the method. The achieved SRR improvement is no less than 2 dB. The method enables effective extraction of target echoes and suppression of reverberation in strong reverberation backgrounds, demonstrating good practical application prospects.

5. Conclusion

This paper addressed the challenge of strong reverberation interference suppression in underwater acoustic signal processing by proposing a sparse dictionary construction method based on multi-order FRFT domain feature fusion. Centered on the core concept of 'multi-order extraction + weighted fusion + sparse reconstruction,' the method achieves effective extraction of target echo signals and suppression of interference. Processing and analysis of measured data demonstrated that the target echo in the output signal was effectively enhanced. Under conditions of low initial SRR ranging from 0 dB to 5 dB, the method consistently achieved an SRR improvement of no less than 2.1 dB, with a maximum improvement of 15.6 dB. These results fully illustrate the method's capability to enhance SRR and suppress interference in strong reverberation backgrounds, as well as its adaptability and robustness to complex underwater acoustic environments. The proposed method not only theoretically extends the application boundaries of the FRFT within sparse representation but also provides an effective technical approach for addressing strong interference suppression issues in underwater acoustic detection and recognition systems.

Fundings

This work was supported by the Joint National Natural Science Foundation of China (no. U22A2044) and the Key Laboratory Fund from Underwater Test and Control Technology (no. 2023-JCJQ-LB-030).

Conflict of interest

The authors declare that there are no known competing financial interests or personal relationships that could have influenced the work described in this paper.

Authors' contribution

Tongjing Sun: data curation, formal analysis, investigation, resources, writing – review and editing, funding acquisition, supervision. Lei Chen: conceptualization, data curation, investigation, methodology, writing – original draft. Xiaohong Deng: data curation, formal analysis, writing – review and editing. All authors reviewed and approved the final manuscript.

Acknowledgments

This article uses experimental data collected at Shanghai Jiaotong University (China), and we gratefully acknowledge our colleagues for their experimental skills.

References

1. Cui X., Chi C., Li S., Li Z., Li Y., Huang H. (2023), Waveform design using coprime frequency-modulated pulse trains for reverberation suppression of active sonar, *Journal of Marine Science and Engineering*, **11**(1): 28, https://doi.org/10.3390/jmse11010028.
2. Deng B., Tao R., Qi L., Liu F. (2005), A study on anti-reverberation method based on fractional Fourier transform, *Acta Armamentarii*, **26**(6): 761–765, https://doi.org/10.3321/j.issn:1000-1093.2005.06.010.
3. Fan Y. (2024), *Parameter adaptive setting and global optimization improvement for gradient descent methods*, Master's thesis, Northwest Normal University.
4. Ge F., Zhang Y. (2023), Review of weak underwater acoustic signal processing, *Journal of Signal Processing*, **39**(10): 1728–1747, https://doi.org/10.16798/j.issn.1003-0530.2023.10.002.
5. Lei L. (2014), *Sparse representation of signals via wavelet modulus maxima and compressed sensing reconstruction*, Master's thesis, Beijing Jiaotong University.
6. Liang Q., Hu P., Chen Z., Li J. (2024), Frequency domain matching filter detection method based on FRFT, *Ship Science and Technology*, **46**(05): 69–73.
7. Liao M., Zhang X., Zhang X. (2014), Classification and recognition of underwater acoustic signal based on sparse representation, *Journal of Detection & Control*, **36**(04): 67–70+77.
8. Liu D., Shi G., Zhou J. (2008), New method for signal sparse decomposition over a redundant dictionary, *Journal of Xidian University*, pp. 228–232.
9. Liu G., Zhou X. (2018), Interfere suppression and communication signal reconstruction method based on signal sparse representation in complex electromagnetic environment, *Computer Systems & Applications*, **27**(11): 149–154, https://doi.org/10.15888/j.cnki.csa.006613.
10. Ma J., Miao H., Su X., Gao C., Kang X., Tao R. (2018), Research progress in theories and applications of the fractional Fourier transform, *Opto-Electronic Engineering*, **45**(06): 5–28, https://doi.org/10.12086/oee.2018.170747.
11. Meng W. (2024), *Research method of underwater acoustic direction of arrival estimation based on noise integral sparse Bayesian learning*, Master's thesis, South China University of Technology.
12. Sun T., He J., Gu Y. (2016), Underwater echo signal processing method based on sparse decomposition, *Journal of Data Acquisition and Processing*, **31**(02): 282–288, https://doi.org/10.16337/j.1004-9037.2016.02.007.
13. Wang H. (2020), *Research on active sonar target classification and recognition based on sparse representation*, Master's thesis, Hangzhou Dianzi University.
14. Weng Y. (2020), Detection and sorting algorithm of multi-component LFM signals based on fractional Fourier transform, *Aerospace Electronic Warfare*, **36**(02): 33–37+55, https://doi.org/10.16328/j.htdz8511.2020.02.009.

15. Wu Y., Xing C., Zhang D., Xie L. (2021), A denoising processing method for the low-frequency underwater acoustic signal based on K-SVD, *Journal of Yunnan University of Nationalities (Natural Sciences Edition)*, **30**(04): 387–393.

16. Yang J. (2023), The application development of sonar technology in marine resource exploration, *Audio Engineering*, **47**(12): 27–29, https://doi.org/10.16311/j.audioe.2023.12.006.

17. Yu G., Park S. (2017), Multipe targets detection in strong reverberation environments, *Journal of Huazhong University of Science and Technology (Natural Science Edition)*, **45**(03): 89–93, https://doi.org/10.13245/j.hust.170316.

18. Zhang Z., Yang X., Dai W. (2014), Pick-up of echo starting location in reverberation, *Ship Electronic Engineering*, **34**(11): 73–75+79.

19. Zhou Z., Wu J., Yuan C., Bai M., Gui Z. (2023), A novel K-SVD dictionary learning approach for seismic data denoising, *Oil Geophysical Prospecting*, **58**(05): 1072–1083, https://doi.org/10.13810/j.cnki.issn.1000-7210.2023.05.005.