ARCHIVES OF ACOUSTICS
3, 4, 231-248 (1978)

A METHOD OF PREDICTING NOISE EQUIVALENT LEVEL VALUE IN URBAN STRUCTURE

RUFIN MAKAREWICZ, GABRIELA KERBER

The Chair of Acoustics, Adam Mickiewicz University
(Poznań, 60-769 Poznań, ul. Matejki 48/49)

The paper presents a method of predicting the noise equivalent level value in urban structure. Unlike the methods used thus far, the presented method consists in determination of a parameter a which is a measure of the energy reaching the observation point during motion of a single source. The method is very laborious and, therefore, the paper presents a solution of the problem of determination of the minimum number of the measurements of the parameter a, necessary to determine the noise equivalent level value with a preset accuracy.

1. Introduction

The pooling investigations performed among the inhabitants of towns [13, 23] have revealed that they rate the traffic noise as the most nuisant type of noise. Within the already existing urban structure, where it is impossible to change the localization of buildings and transportation routes, the noise can be minimized by an appropriate "programming" of the structure and intensity of transportation traffic. In such a case it is necessary to know the relation between the parameters describing acoustic field (noise evaluation indicators, e.g. noise equivalent level L_{eq}) [1] and the value of "traffic intensity" (n_i) of the noise sources, i.e. transportation vehicles. Provided the relation $L_{eq} = f(n_i)$ is available, it is feasible to determine the permissible values of traffic intensity which correspond to a predetermined (e.g. given by a standard) value of L_{eq}.

Determination of the function $f(n_i)$ in an urban area is much more difficult than in the case of an open area, e.g. in the vicinity of a motor way where the noise sources move at a constant speed (an extensive reference list ot this problem is given in [16]).

---

[1] Other noise evaluation indicators are discussed in [16].

The configuration of acoustic field in an urban area complicates as a result of reflections from many planes and non-uniform motion of noise sources (rounda-bouts, crossings, traffic lights etc). Only if urban structure is highly symmetric (e.g. "tunnel" development), it is possible to determine the function $f(n_i)$ by mathematical analysis. In other cases this function is being obtained by regression analysis. The results of investigations in the form of plots, nomograms, tables, corrections etc. are presented in [1, 3, 6, 7, 8, 24].

This paper presents the principle (Section 3) and experimental verification (Section 4) of a new method of predicting the noise equivalent level value in an urban area. The theoretical background of this method was discussed in [16].

The essential difference between this method and the presently used method based on regression analysis is that the parameters α_i appearing in function

$$L_{eq} = 10 \log\Big(\sum_i n_i \alpha_i + C\Big)$$

are determined from the measurement of the signal emitted by a single source (Section 3.2) rather than from the set of values L_{eq} for the resultant signal (being the sum of signals coming from individual sources).

The problem of minimization of the number of measurements, necessary to achieve the required degree of consistency of theoretical and experimental results, is discussed in Section 5.

The basic concept of the new method of noise equivalent level determination results from equation (13) derived in [16] from the definition (1) of noise equivalent level. The consistency of the proposed method with other measurement methods is demonstrated in Section 2 by deriving from the same definition equations (9) and (12) encountered in the literature and thus proving their equivalence.

2. Methods of equivalent level measurement

The noise equivalent level L_{eq} is defined [20] as

$$L_{eq} = 10 \log \frac{1}{T} \int_{-T/2}^{T/2} 10^{0,1L(t)}\, dt, \tag{1}$$

where $L(t)$ is the noise level measurement in dB(A) and T – measurement duration.

There exist instruments like Brüel-Kjaer, type 4426, or RFT, type 00005, which measure L_{eq} directly.

The value of noise equivalent level L_{eq} can also be determined by using a noise level meter and a recording voltmeter to register the course of $L(t)$.

By dividing the noise level into classes of constant width $(L_i, L_i + \Delta L)$

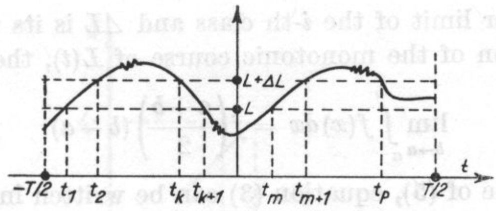


Fig. 1. Changes of sound level vs. time $L(t)$ [dB(A)]

(cf. Fig. 1), the integral in definition (1) can be presented in the form of a sum,

$$\int_{-T/2}^{T/2} 10^{0,1L(t)}dt = \sum_{j=1}^{N} \int_{t_j}^{t_{j+1}} 10^{0,1L(t)}dt, \tag{2}$$

where $t_1 = -T/2$, $t_{N+1} = T/2$ are the instants of the beginning and the end of the measurement, respectively.

By approximating each integral by the product of the mean value of the integral function in the interval (t_j, t_{j+1}) and the length $t_{j+1}-t_j$ of this interval, we obtain

$$\int_{-T/2}^{T/2} 10^{0,1L(t)}dt = \sum_{j=1}^{N} 10^{0,1L_j}(t_{j+1}-t_j), \tag{3}$$

where

$$10^{0,1L_j} = \frac{1}{t_{j+1}-t_j}\int_{t_j}^{t_{j+1}} 10^{0,1L(t)}dt. \tag{4}$$

When the width of the class tends to zero ($\Delta L \to 0$), the length $t_{j+1}-t_j$ of the integration interval tends also to zero and thus equation (4) can be written in the form

$$10^{0,1L_j} \approx 10^{0,1(L_i+\frac{1}{2}\Delta L)}, \tag{5}$$

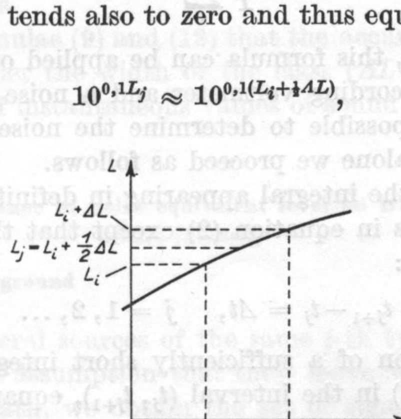


Fig. 2. Noise level L_j (equation (3)) as the mean value of values L_i, $L_i+\Delta L$ defining the lower and the upper limit of a "class"

where L_i is the lower limit of the i-th class and ΔL is its width (Fig. 2) since, under the assumption of the monotonic course of $L(t)$, the relation

$$\lim_{b\to a} \int_a^b f(x)\,dx = f\left(\frac{a+b}{2}\right)(b-a) \tag{6}$$

is satisfied. By virtue of (5), equation (3) can be written in the following form:

$$\int_{-T/2}^{T/2} 10^{0,1L(t)}\,dt = \sum_{j=1}^{N} 10^{0,1(L_i+\Delta L/2)}(t_{j+1}-t_j). \tag{7}$$

It may happen that the course of $L(t)$ is within the same class $(L_i, L_i+\Delta L)$ for several time intervals (t_j, t_{j+1}), as for the intervals (t_1, t_2), (t_k, t_{k+1}), (t_m, t_{m+1}) in Fig. 1. This means that the sum (7) may contain several terms with identical factor $10^{0,1(L_i+\Delta L/2)}$. By grouping such terms and next summing with respect to all classes we obtain

$$\int_{-T/2}^{T/2} 10^{0.1L(t)}\,dt = \sum_i t_i \cdot 10^{0.1(L_i+\Delta L/2)}, \tag{8}$$

where

$$t_i = \sum_i (t_{j+1}^{(i)} - t_j^{(i)})$$

is the total time interval in which the temporary value of noise level $L(t)$ satisfies the unequality $L_i < L(t) < L_i + \Delta L$.

By using equation (8) in definition (1), we obtain an expression determining the value of noise equivalent level L_{eq} in terms of instantaneous values of sound level $L(t)$ measured in A decibels:

$$L_{eq} = 10 \log \frac{1}{T} \sum_i t_i \cdot 10^{0.1(L_i+\Delta L/2)}. \tag{9}$$

As it was mentioned, this formula can be applied only if the measurement is performed using a recording voltmeter and a noise level meter. To derive the formula making it possible to determine the noise equivalent level value using noise level meter alone we proceed as follows.

Let us assume that the integral appearing in definition (1) can be replaced by a sum of integrals as in equation (2) except that the integration intervals are equal to each other:

$$t_{j+1} - t_j = \Delta t, \quad j = 1, 2, \ldots$$

Under the assumption of a sufficiently short integration interval Δt and monotonic course of $L(t)$ in the interval (t_j, t_{j+1}), equation (2) can be written in the form

$$\int_{-T/2}^{T/2} 10^{0.1L(t)}\,dt = \Delta t \sum_{j=1}^{N} 10^{0.1L_j}, \tag{10}$$

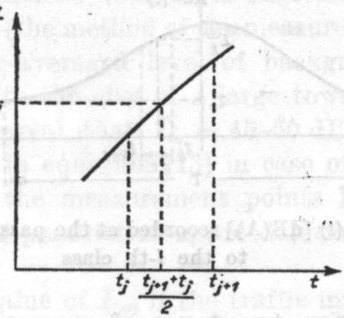


Fig. 3. Noise level L_j (equation (10)) corresponding to the moment $t = \frac{1}{2}(t_j + t_{j+1})$.

where $L_j = L((t_{j+1} + t_j)/2)$ is the value of the sound level at the instant $\frac{1}{2}(t_{j+1} + t_j)$ (cf. Fig. 3).

Since $T = N\Delta t$, by virtue of equation (10) we obtain from definition (1) another expression for calculating the noise equivalent level value:

$$L_{\rm eq} = 10 \log \frac{1}{N} \sum_{j=1}^{N} 10^{0.1 L_j}. \tag{11}$$

If the successive readings of level L are repeated in such a way that values L_j correspond to numbers n_j, then

$$L_{\rm eq} = 10 \log \frac{1}{N} \sum_{j} n_j \cdot 10^{0.1 L_j}. \tag{12}$$

This formula makes it possible to determine the noise equivalent level value by reading instantaneous values of sound level (expressed in dB(A)) in equal time intervals.

It follows from formulae (9) and (12) that the accuracy of $L_{\rm eq}$ determination is the higher the smaller the width of the class ($\Delta L \to 0$) is and the more frequently the readings of instantaneous values of sound level are taken, i.e. when $\Delta t \to 0$.

3. Dependence of noise equivalent level on transportation traffic intensity

3.1. Theoretical background

Let us consider several sources of the same j-th type (e.g. 7 vehicles of the same type). Under the assumption that they move with the same speed $V_k(t)$ along the same l-th path, we obtain the set of signals $L^{(i)}(t)$ very similar to each other (cf. Fig. 4).

To simplify the notation, we introduce an index i for each possible combination $(j\,k\,l)$ which will identify the "class" of the source [16].

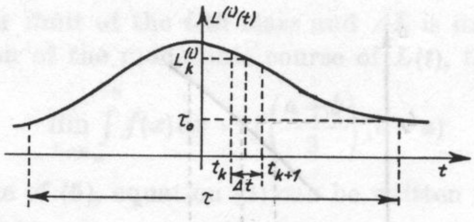


Fig. 4. Noise level changes $L^{(i)}(t)$ [dB(A)] recorded at the passage of a single source belonging to the i-th class

If we have, for instance, two types of sources — two types of vehicles ($j = 1, 2$) which move at the same speed $V_k(t)$ ($k = 1$) along two different paths ($l = 1, 2$), we shall register four different signals $L^{(i)}(t)$ ($i = 1, 2, 3, 4$) since this is just the number of possible combinations among the indices ($j\,k\,l$).

The urban noise is a result of the superposition of signals produced by individual sources. As it has been demonstrated [16] the noise equivalent level L_{eq} can be determined by means of the formula

$$L_{eq} = 10\log\frac{1}{I_0}\Big[\sum_{i=1}^{M} n_i \int_{-\infty}^{+\infty} I_i(t)\,dt + \bar{I}^0\Big],$$

where $I_i(t)$ is the time-dependent intensity of noise generated by a moving source of i-th class, $\bar{I}^0$ — the average background intensity, I_0 — the reference intensity, n_i — the number of sources of i-th class passing the observation point per unit time (traffic intensity), M — the number of classes of noise sources.

Actually it is the pressure level which is the measurable quantity

$$L(t) = 10\log p^2(t)/p_0^2,$$

where $p_0 = 2\cdot 10^{-5}\,N/\mathrm{m}^2$. As it was demonstrated by Beranek [2], it can by assumed that the pressure level equals to intensity level with an accuracy much better than 1 dB,

$$L(t) = 10\log\frac{I(t)}{I_0},$$

where $I_0 = 10^{12}\,\mathrm{W/m}^2$.

Hence

$$L_{eq} = 10\log\Big[\sum_{i=1}^{M} n_i a_i + 10^{0.1\bar{L}_0}\Big], \tag{13}$$

where

$$a_i = \int_{-\infty}^{+\infty} 10^{0.1L_i(t)}\,dt \tag{14}$$

and $L_i(t)$ is the time-dependent noise level expressed in dB(A), produced by a single source of i-th class (the method of the measurement of a_i will be discussed later on); $\bar{L}_0$ is the time-averaged level of background intensity $-\bar{I}^0$. The measurements performed in the city of a large town at a large distance from the stream of vehicles reveal that $\bar{L}_0 \approx 45$–50 dB(A). This allows to omit the last term of the sum in equation (13) in case of the high traffic intensity (large values of n_i) and the measurement points located close to the road.

Provided the values of parameters a_i are determined in advance, equation (13) makes it possible:

(a) to determine the value of L_{eq} if the traffic intensity n_i is known;

(b) to modify the value of L_{eq} by imposing appropriate limitations upon the traffic structure: $n_i \leqslant n_i^*$.

These problems will be treated in more detail later on.

3.2. Method of calculating the values of a_i

Equation (14) provides a way of calculating the values of a_i. As it was already mentioned, the quantity $L_i(t)$ appearing in this equation is the noise level generated by a single source of the i-th class. To obtain the course of $L_i(t)$, the measurement should be performed under the conditions when only a single source is actually moving in the vicinity of observation point. It is thus convenient to perform such measurements in the night (preliminary investigations indicate that the method of model measurements can also be applied for this purpose).

However, such measurement yields the signal $L_j^{(i)}(t)$ (noise level produced by the source and the background) rather than "pure" signal $L_i(t)$, and the former is different for each j-th source belonging to the same i-th class:

$$L_j^{(i)}(t) = 10\log\left(\frac{I_j^{(i)}(t)}{I_0} + \frac{\tilde{I}^{(0)}}{I_0}\right).$$

Introducing the notation

$$L_j^{(i)}(t) = 10\log\frac{I_j^{(i)}(t)}{I_0}, \qquad \tilde{L}_0 = 10\log\frac{\tilde{I}^{(0)}}{I_0}$$

($L_j^{(i)}$ is the noise level produced by the source only, $\bar{L}_0$ — the noise level associated with the acoustic background present during the measurement), from (14) we obtain

$$a_{ij} = \int_{-\tau/2}^{\tau/2} [10^{0.1L_j^{(i)}(t)} - 10^{0.1\tilde{L}_0}]\,dt, \tag{15}$$

where τ is the signal duration (cf. Fig. 4), i.e. the time of source passage, $L_j^{(i)}(t)$ — the registered change of intensity level with time, $\tilde{L}_0$ — the background level during the measurement. The quantity $\tilde{L}_0$ has, as a rule, a value different to that of $\bar{L}_0$ (equation (13)) (for instance, the acoustic background in the night, when the measurements of $L^{(i)}(t)$ are performed, is different than during the day).

To determine the values of parameters a_{ij} we divide, in agreement with equation (15), the interval $(-\tau/2, \tau/2)$ into equal intervals Δt and obtain the formula

$$a_{ij} = \Delta t \sum_{k=1}^{M} 10^{0.1 L_{jk}^{(i)}} - \tau \cdot 10^{0.1 \bar{L}_0}, \tag{16}$$

where $L_{jk}^{(i)}$ is, in agreement with Fig. 4, the value of the sound level at the instant $t^{(k)} = \frac{1}{2}(t_{k+1} + t_k)$, i.e. in the middle of the interval (t_{k+1}, t_k).

4. Measurement results

4.1. Classification of noise sources

The parameters a_{ij} were determined for vehicles moving along a two-way street. The measurement point was placed at a height of the third floor at a distance of 30 m from the nearest street crossing. Fig. 5a presents a cross-section of the street and Fig. 5b – its view from the top with the marked measurement point A.

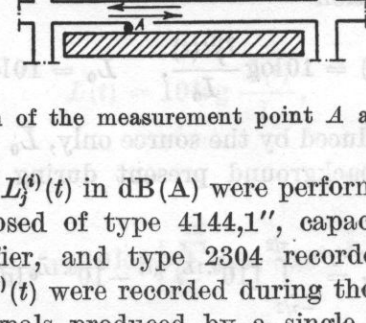


Fig. 5. Localization of the measurement point A at a two-way street

The measurements of $L_j^{(i)}(t)$ in dB(A) were performed using a set of Brüel-Kjaer equipment composed of type 4144,1″, capacitance microphone type, 2602 microphone amplifier, and type 2304 recorder with 50 dB potentiometer. The courses of $L_j^{(i)}(t)$ were recorded during the night between 23h and 3h since undisturbed signals produced by a single vehicle were possible to obtain only during that time. Typical courses of the noise level $L_j^{(i)}(t)$ [dB(A)] produced by Fiat motor-cars and Jelcz buses moving to the right and to the left are shown in Fig. 6.

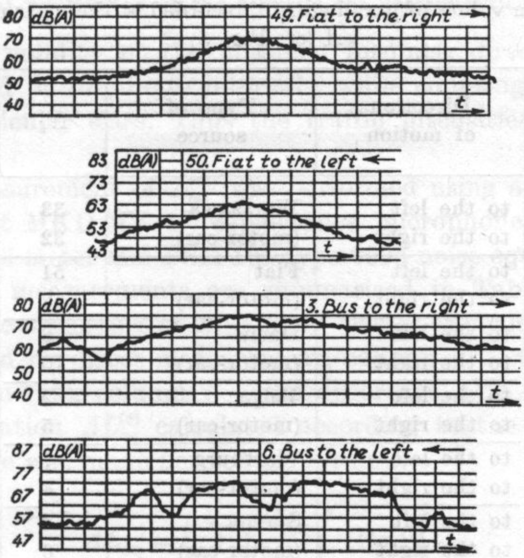


Fig. 6. Noise level changes $L_j^{(i)}(t)$ [dB(A)] for several single vehicles

The values of parameters a_{ij} for each course of $L_j^{(i)}(t)$ were calculated using equation (16) and next mean values a_i were determined from the formula

$$a_i = \frac{1}{N_i} \sum_{j=1}^{N_i} a_{ij}, \tag{17}$$

where N_i is the number of registered signals produced by the sources of the i-th class.

The values of a_i and the numbers N_i for 22 classes of sources are given in Table 1. It follows from this Table that the values a_i corresponding to buses are higher by an order of magnitude from the remaining values. One can also see a marked difference between the values of a_i corresponding to different directions of the vehicle movement. This difference is associated with a different distance from the measurement point. Therefore, for further calculations, we accept a 4-class classification of the noise sources:

1. motor-cars moving to the right,
2. motor-cars moving to the left,
3. buses moving to the right,
4. buses moving to the left.

The mean values of a_i calculated from equation (17) for these classes of noise sources amount in hour/veh., respectively, to:

$$a_1 = 2\,359, \quad a_2 = 1\,072, \quad a_3 = 49\,049, \quad a_4 = 28\,397.$$

Table 1. Mean values of parameters a_i recorded at the measurement point A (cf. Fig. 5)

No. of source class	Direction of motion	Type of source	N_i	a_i $\left[\frac{\text{hour}}{\text{veh.}}\right]$
1	to the left	Warszawa	33	1 091
2	to the right	(motor-car)	32	2 004
3	to the left	Fiat	51	1 166
4	to the right	(motor-car)	52	2 607
5	to the left	Dacia	8	942
6	to the right	(motor-car)	10	2 159
7	to the left	Volga	2	1 281
8	to the right	(motor-car)	5	1 755
9	to the left	Wartburg	7	783
10	to the right	(motor-car)	4	1 392
11	to the left	Syrena	4	900
12	to the right	(motor-car)	6	3 546
13	to the left	Moskvich	2	379
14	to the right	(motor-car)	5	3 257
15	to the left	Trabant	1	708
16	to the right	(motor-car)	3	2 883
17	to the left	Skoda	3	1 554
18	to the right	(motor-car)	2	1 402
19	to the left	Nysa	6	776
20	to the right	(pick-up)	2	422
21	to the left	Jelcz	13	28 397
22	to the right	(bus)	10	49 049

The standard deviations Δa_i calculated from equation

$$\Delta a_i = \left\{\frac{1}{N_i - 1}\sum_{j=1}^{N_i}(a_{ij} - a_i)^2\right\}^{1/2} \tag{18}$$

amount to:

$$\Delta a_1 = \pm 1555, \quad \Delta a_2 = \pm 723, \quad \Delta a_3 = \pm 18732, \quad \Delta a_4 = \pm 10721.$$

Substituting the mean values of a_i to equation (13), we obtain the expected value of the noise equivalent level at measurement point A in the street of the cross-section shown in Fig. 5,

$$L_{\text{eq}} = 10\log(2359 n_1 + 1072 n_2 + 49049 n_3 + 28397 n_4 + 10^{0.1\bar{L}_0}), \tag{19}$$

where n_1 is the number of motor-cars moving to the right [veh./hour], n_2 – number of motor-cars moving to the left [veh./hour], n_3 – number of buses moving to the right [veh./hour], n_4 – number of buses moving to the left [veh./hour], $\bar{L}_0$ – acoustic background.

At high traffic intensity the last term accounting for acoustic background can be disregarded in calculations.

4.2. Experimental verification of the formula for determination of L_{eq}

To verify the validity of the obtained formula, direct measurements of L_{eq} were performed in 10-minute intervals, while counting simultaneously the vehicles of a particular class. Thus the traffic intensities n_1, n_2, n_3, n_4 were determined.

The direct measurement of $L_{eq}^{(m)}$ was performed using a set of RFT equipment consisting of MKD MV 1″ capacitance microphone No 3103, PSI 202 precision noise level meter and PSM 101 type 0005 noise equivalent level meter.

The results of measurements are summarized in Table 2. Columns 1 to 4 present the traffic intensities of individual classes of vehicles, column 5 — the value $L_{eq}^{(m)}$ obtained by direct measurement, column 6 — the value $L_{eq}^{(c)}$ obtained from equation (19), column 7 — the difference $L_{eq}^{(m)} - L_{eq}^{(c)}$, column 8 — the standard deviation $\Delta L_{eq}^{(c)}$ calculated according to the law of "error propagation" [10]. Use was made here of the formula

$$\Delta L_{eq} = \left\{ \sum_{i=1}^{4} \left(\frac{\partial L_{eq}}{\partial a_i} \right)^2 (\Delta a_i)^2 \right\}^{1/2},$$

i.e.

$$\Delta L_{eq}^{(c)} = \frac{10 \log e}{\sum_{i=1}^{4} n_i a_i + 10^{0,1 \bar{L}_0}} \left\{ \sum_{i=1}^{4} n_i^2 (\Delta a_i)^2 \right\}. \tag{20}$$

Table 2. Summary of the values of L_{eg} calculated and measured at the measurement point A (cf. Fig. 5) at known traffic intensities n_i

No	n_1 [veh./hour]	n_2 [veh./hour]	n_3 [veh./hour]	n_4 [veh./hour]	$L_{eq}^{(m)}$ [dB(A)]	$L_{eq}^{(c)}$ [dB(A)]	$L_{eq}^{(m)} - L_{eq}^{(c)}$ [dB(A)]	$\Delta L_{eq}^{(c)}$ [dB(A)]
	1	2	3	4	5	6	7	8
1	174	210	6	12	61.3	61.0	0.3	1.2
2	144	156	18	12	62.8	62.4	0.4	1.1
3	174	120	12	6	61.5	61.1	0.4	1.2
4	132	84	6	12	60.5	60.2	0.3	1.1
5	126	216	18	6	62.5	62.0	0.5	1.2
6	168	174	18	12	63.0	62.6	0.4	1.1
7	108	120	6	12	60.5	60.1	0.4	1.3
8	162	144	12	12	62.1	61.7	0.4	1.6
9	120	102	6	6	59.7	59.0	0.7	1.3
10	162	84	12	12	61.9	61.5	0.4	1.1
11	162	180	12	18	62.6	62.2	0.4	1.1
12	198	156	6	—	59.9	59.7	0.2	1.6
13	186	198	6	12	61.3	61.1	0.2	0.9
14	150	222	12	12	62.2	61.8	0.4	1.1
15	114	138	6	6	59.8	59.5	0.3	1.2
16	150	120	12	18	62.4	62.0	0.4	1.1
17	174	138	6	6	60.4	60.1	0.3	1.3

It follows from Table 2 that the difference $L_{eq}^{(m)} - L_{eq}^{(c)}$ between the directly measured value and the value obtained from equation (19) for given traffic intensities n_1, n_2, n_3, n_4 is smaller than 0.7 dB(A). The average value of this difference equals to 0.4 dB(A).

It may be thus concluded that the method of predicting the noise equivalent level L_{eq} in urban structures, based on equation (13), is fully justified.

5. Optimization of the measurement process

The spectacular agreement between the values L_{eq} obtained by calculation and by direct measurement can be explained as due to very small differences between the values $a_i (i = 1, 2, 3, 4)$ appearing in (19) and the values

$$\bar{a}_i = \lim_{N_i \to \infty} a_i$$

which apply to the whole population of noise sources passing the observation point. Let us denote these differences by δa_i. The value of δa_i is the smaller the more accurately the value of a_i in (17) is calculated, i.e. the larger is the number of measurements N_i.

The question arises how large should be the number N_i to make the values of δa_i small enough to determine the value of L_{eq} from equation (13) with an error δL_{eq} smaller than k dB(A) with a probability p.

Replacing the standard deviations Δa_i in (20) by the quantities δa_i, we obtain a general expression defining the error δL_{eq},

$$\delta L_{eq} = \frac{10 \log e}{\sum_{i=1}^{M} n_i a_i + 10^{0.1 \bar{L}_0}} \left\{ \sum_{i=1}^{M} n_i^2 (\delta a_i)^2 \right\}^{1/2},$$

where M is the number of classes of noise sources.

From the requirement $\delta L_{eq} < k$ dB and under the assumption that every class provides the same "contribution" to the value δL_{eq}, we obtain

$$\delta L_{eq} < \frac{k}{\sqrt{M}} \frac{\sum_{i=1}^{M} n_i a_i + 10^{0.1 \bar{L}_0}}{n_i 10 \log e}. \tag{21}$$

The parameters a_i in (17) are random values.

If a_{ij} $(j = 1, 2, \ldots, N_i)$ have a normal distribution [10], then

$$\delta a_i = \frac{t_p^{(i)} \Delta a_i}{\sqrt{N_i}}, \tag{22}$$

where $t_p^{(i)}$ is the value obtained from Student's distribution for the probability p at $N_i - 1$ degrees of freedom, Δa_i – standard deviation given by (18), N_i – the number of measurements of $a_{ij} (j = 1, 2, \ldots, N_i)$.

From equations (21) and (22) we find N_i^* — the number of measurements of parameters $a_{ij}(j = 1, 2, \ldots, N_i)$, necessary to reduce the error δL_{eq} of the disscussed method below k dB with a probability p:

$$N_i^* = M\left(\frac{10\log e\, t_p^{(i)} n_i \Delta a_i}{k\sum_{i=1}^{M} n_i a_i + 10^{0.1\bar{L}_0}}\right)^2. \tag{23}$$

To find the value of N_i^* from this formula a "pilot" series of measurements of $a_{ij}(i = 1, 2, \ldots, M; j = 1, 2, \ldots, N_i)$ should be performed and the values of a_i (17) Δa_i(18), and $t_p^{(i)}$ should be determined. If the inequality $N_i^* < N_i$ is satisfied, there is no need to perform additional measurements since the power of the set $\{a_{ij}\}$ is adequate.

It follows that the number of measurements N_i^*, necessary to determine the value of L_{eq} from (13) with an error less than k dB (with probability p), tends to zero if the intensity of the motion of noise sources tends to zero.

Equation (23) has been derived under the assumption that the partition of noise sources into individual classes has been executed "a priori". Practically it happens that the values corresponding e.g. to various types of noise sources do not differ much from each other and the need to form separate classes for these sources becomes questionable.

Table 1 presents mean values of a for various types of motor-cars. In comparison with the values of a for buses the differences between these values are small and, therefore, we have decided to form a single common class for all motor-cars regardless their types and taking into consideration only their direction of motion.

In general case this problem can be formulated as follows: how to perform a partition of all sources into classes in such a way that

(a) The error δL_{eq} of determining the value of equivalent level L_{eq} from (13) is less than k dB with a probability p.

(b) The total number of the measurements of parameters $a_{ij}(j = 1, 2, \ldots, N_i,\ i = 1, 2, \ldots, M)$ is minimum, i.e.

$$\sum_{i=1}^{M} N_i^* = \text{minimum}. \tag{24}$$

Condition (b) can be regarded as the aim of the optimization of the measurement process.

Similarly to the case of determination of the "necessary" number of measurements N_i^*, let us assume that we have at our disposal a "pilot" series of measurements $\{a\}$. As a first step to solving the above problem we introduce a partition into "unquestionable" classes, i.e. the classes with markedly different values of a. Let the number of these classes be M_1. It follows from the example discussed in Section 4 that $M_1 = 2$ if the noise sources have to be divided into at least two classes: motor-cars and buses. It is less evident whether it is necessary to split these classes with respect to different motion directions "to the left" and "to the right".

Next, we check, using equation (23), whether the number of measurements $N_{i,1}$ in each class is sufficient (at the assumed value of probability p and error k), i.e. whether the relation $N_{i1} > N_{i,1}^*$ is satisfied.

In turn, we perform a new partition of the set of parameters $\{a_{ij}\}$ into M_2 classes ($M_2 > M_1$), check the validity of relation $N_{i,2} > N_{i,2}^*$, repeat the same for M_3 classes etc.

At each q-th partition into various classes, i.e. when the set $\{a_{ij}\}$ (of power N) is divided into subsets of power $N_{i,q}$, use can be made of the papers of Christie, Hillquist and Scott, Jonasson, Lewis, Olson, Nelson and Piner, Priede, Rathe, Ulrich, Waters [4, 5, 9, 11, 15, 17, 19, 21, 22, 25, 26, 27] which give values of the noise level as functions of velocity and acceleration.

Further on we consider only those partitions into classes (among all $q = 1, 2, \ldots$) for which the inequalities

$$N_{i,q} > N_{i,q}^*, \quad i = 1, 2, \ldots, M_q, \tag{25}$$

equivalent to condition (a), are satisfied. (If none of the partitions into classes satisfies these inequalities, then the set $\{a_{ij}\}$ has to be supplemented with additional measurements of parameter a.)

Out of all partitions into classes, which satisfy inequalities (25), we choose that one for which the sum of necessary measurements in each class $N_{i,q}^*$ ($i = 1, 2, \ldots, M_q$) satisfies condition (24).

This partition into classes in the optimum one, since it permits the value of equivalent level (13) to be determined with an error less than k dB (with the probability p) from the minimum number of measurements.

Example. Let us assume that measurements of the parameter a (Table 1) are the "pilot" series with the power of individual classes ($M = 4$) $N_1 = 121$, $N_2 = 117$ (classes of motor-cars moving "to the right" and "to the left") and $N_3 = 10$, $N_4 = 13$ (classes of buses moving "to the right" and "to the left").

Let us assume the probability $p = 0.99$ and the value of acceptable error $k = 0.5$ dB (it means that the values L_{eq} obtained from (13) will bear an error less than 0.5 dB with the above probability). From Student's distribution for the numbers of freedom degrees $N_1 - 1 = 120$, $N_2 - 1 = 116$, $N_3 - 1 = 9$, $N_4 - 1 = 12$) we obtain that $t_{0.99}^{(1)} = 2.58$, $t_{0.99}^{(2)} = 2.58$, $t_{0.99}^{(3)} = 3.25$, $t_{0.99}^{(4)} = 3.05$. The values of standard deviations Δa_i are given in Section 4.1. It follows from (13) that the sum in the nominator of expression (23) can be presented in the form

$$\sum n_i a_i + 10^{0.1\bar{L}_0} = 10^{0.1L_{eq}}.$$

Assuming the mean traffic intensities, obtained from the measurements, presented in Table 2: $n_1 = 153$, $n_2 = 151$, $n_3 = 10$, $n_4 = 10$, and the corresponding value $L_{eq} = 61$ dB(A), we have $N_1 = 70$, $N_2 = 15$, $N_3 = 69$, $N_4 = 20$.

The results obtained indicate that to use formula (19) for the determination of the value of L_{eq} with the accuracy 0.5 dB (with the probability $p = 0.99$) the number of measurements a_{ij} in classes 1 and 2 was too large ($N_i^* < N_i$) and in classes 3 and 4 – too small ($N_i^* > N_i$). The two latter classes should be complemented with additional measurements to satisfy condition $N_i^* \leqslant N_i$.

6. Conclusions

The work is primarily aimed at verification of the method of predicting the value of noise equivalent level L_{eq} outlined in [16]. Good agreement of the experimental data and the results obtained from formula (13) confirms the validity of the method (Section 4.2.).

To demonstrate consistency of the new method with other measurement methods, definition (1), which is also the starting point to derive equation (13), is used in Section 1 to derive expressions enabling L_{eq} to be determined either from the record of noise level changes in time (equation (9)) or from a set of instantaneous values of noise level, recorded at equal time intervals (equation (12)).

The method of determining the value of L_{eq} in urban structure follows from equation (13). An application of this formula requires a partition of all sources into appropriate classes. This partition is made basing on the value of parameter a (equations (14) and (15)) which is a measure of the energy reaching the observation point during motion of an individual source. Equation (13) contains the quantities a_i – mean values for particular classes. The error introduced by calculating L_{eq} from (13) is the smaller the more accurately is the value a_i of (17) calculated and the larger is the number of measurements of a_{ij} ($j = 1, 2, \ldots, N_i$). Equation (19) is the form of (13) specified for a particular case.

The problem of minimizing this error is linked to the problem of optimizing the measurements. It was demonstrated in Section 5 that if the value of the error is assumed to be k dB with the probability p, then – after performing a pilot series of measurements of $\{a\}$ for randomly chosen noise sources – the whole population of sources can be divided into classes (after eventual complementing the number of measurements to satisfy the inequality $N_i^* < N_i$) in such a way that the assumed requirements of the accuracy of the method will be satisfied.

The values of parameters a_i depend on the power of acoustic sources (e.g. on the types of cars), the trajectory, the way of operation and the type of urban structure and, therefore, equation (19) is valid only for measurement point A, as indicated in Fig. 5. Therefore the general form of equation (19) is as follows:

$$L_{\rm eq}^{(m)} = 10\log\Big(\sum_{i=1}^{M^{(m)}} n_i^{(m)}\alpha_i^{(m)} + 10^{0.1\bar{L}_0^{(m)}}\Big), \qquad m = 1, 2, \ldots, \tag{26}$$

where $n_i^{(m)}$ is the traffic intensity of noise sources passing the m-th observation point, $M_i^{(m)}$ — number of classes of sources in the vicinity of the m-th point, $\bar{L}_0^{(m)}$ — mean background level measured at the m-th point.

If we assume that the condition of favourable acoustic climate in the vicinity of each of these points is given by inequality

$$L_{\rm eq}^{(m)} < \tilde{L}_{\rm eq}^{(m)}, \qquad m = 1, 2, \ldots, \tag{27}$$

where $\bar{L}_{\rm eq}^{(m)}$ can be considered as values resulting from various standard values for residential areas, hospitals, schools etc., then equation (26) yields information on the requirements which should be imposed on the values of traffic intensities $\{n_i^{(m)}\}$ along particular streets (Fig. 7) to satisfy inequalities (27).

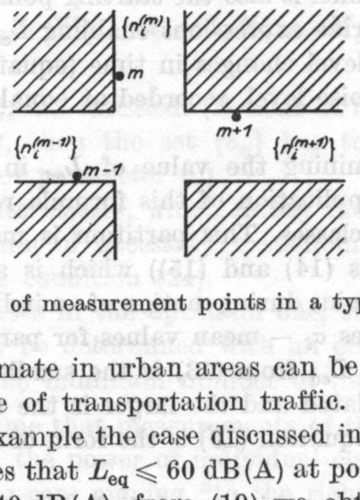


Fig. 7. Localization of measurement points in a typical urban structure

Thus the acoustic climate in urban areas can be formed by suitable "programming" the structure of transportation traffic.

Let us consider for example the case discussed in Section 4. Let us assume that the standard requires that $L_{\rm eq} \leqslant 60$ dB(A) at point A. Assuming the mean background level $\bar{L}_0 = 40$ dB(A), from (19) we obtain

$$2359n_1 + 1072n_2 + 49049n_3 + 28397n_4 < 99\cdot 10^4.$$

This is a particular case of inequality (27) with equation (26) taken into account. This inequality provides information on the maximum intensity of the traffic of motor-cars (n_1 and n_2) and buses (n_3 and n_4) for which the condition of good acoustic climate at point A is still satisfied, i.e. $L_{\rm eq} < 60$ dB(A).

The method of predicting the value of noise equivalent level ($L_{\rm eq}$), presented in this paper, is fairly laborious but the error of $L_{\rm eq}$ determination is very small.

The theoretical and experimental investigations carried out presently at the Chair of Acoustics of the Adam Mickiewicz University are aimed at such development of the presented method which will make it applicable over a possibly widest range of problems. In particular, this method may become a starting point for development of a new methodology of preparing acoustic maps of towns since it permits us not only to determine the acoustic climate (the value of L_{eq}) at various points of the town but also to "program" the structure and intensity (n_i) of traffic in such a way that this climate will satisfy the given requirements, i.e. the values of L_{eq} will be e.g. less than the values directly related to the existing obligatory standards.

References

[1] Anon, *Schallschutz im Stadtebau Richtlinien für die Planung*, DIN 18005, 1969.

[2] L. L. Beranek, *Noise and vibration control*, McGraw Hill. Inc., 4, 35 (1971).

[3] J. S. Bradley, *Prediction and propagation of urban traffic noise levels*, 91-st Meeting of Acoustical Society of America, Washington D. C., 1976.

[4] A. W. Christie et al., *Urban freight distribution: a study of operations in Putney Hight Street*, TRRL Lab. Rep. 556, Crowthorne, 1973.

[5] — *Urban freight distribution: studies of operations in shopping streets at Newbury and Camberley*, TRRL Lab. Rep. 603, Crowthorne, 1973.

[6] L. Czabalay and I. Pinter, *Study of the relation between street noise and density of traffic*, Egészségtudomàny, 18, 229–235 (1974).

[7] B. Favre, *Bruit aux abords des carrefours routiers: résultats d'un programme de simulation*, 9-th ICA, B-33, Madrid, 1977.

[8] D. J. Fisk et al., *Prediction of urban traffic noise*, 8-th Int. Congr. on Acoustics, London, 1974.

[9] R. Hillquist and W. Scott, *Motor vehicle noise spectra: their characteristics and dependence upon operation parameters*, JASA, 58, 2–10 (1975).

[10] D. J. Hudson, *Statistics. Lectures on elementary statistics and probability*, Geneva, 1964.

[11] H. G. Jonasson, *The accuracy of traffic noise prediction*, Dept. of Build. Acoustics, Lund Inst. of Techn., 1975.

[12] K. D. Kryter, *The effects of noise on man*, Academic Press, New York, 1970.

[13] F. J. Langdon, *Noise nuisance caused by road traffic in residential areas*, J. Sound and Vibr., 47, 246–282 (1976).

[14] A. Lawrance, *Stop-start traffic noise*, 7-th ICA, Budapest, 1971.

[15] P. T. Lewis, *The noise generated by single vehicle in freely flowing traffic*, J. Sound and Vibr., 30, 191–206 (1973).

[16] R. Makarewicz, *Theoretical prediction of equivalent noise level*, Archives of Acoustics (to appear).

[17] N. Olson, *Survey of motor vehicle noise*, JASA, 52, 1291–1306 (1972).

[18] G. L. Osipov et al., Gradostroitielnyje miery borby s szumom, Stroizdat, Moskwa, 1975.

[19] P. M. Nelson and R. J. Piner, *Classifying road vehicles for the prediction of road traffic noise*, TRRL Lab. Rep. 752, Crowthorne, 1977.

[20] K. S. Pearson and R. L. Bennet, *Handbok of noise ratings*, NASA, CR-2376, 1974.

[21] T. Priede, *The effect of operating parameters on sources of vehicle noise*, J. Sound and Vibr., 43, 239–252 (1975).

[22] E. J. Rathe, *Survey of the exterior noise of some passenger cars*, J. Sound and Vibr., 29, 483–499 (1973).

[23] J. Sadowski and B. Szudrowicz, *The effect of materials and construction on the acoustic climate of flats and the health of the inhabitants*, Prace Inst. Techn. Bud. Warszawa, 1975 (in Polish).

[24] J. Sadowski, *Acoustics in architecture, urbanistics and building*, Arkady, Warszawa 1971 (in Polish).

[25] S. Ullrich, *Der Einfluss von Fahrzeuggeschwindigkeit und Strassenbelag auf den energieequivalent Dauerschallpegel des Lärmes von Strassen*, Acustica, **30**, 90–99 (1974).

[26] — *Fahrgeräuschpegel von PKW and LKW an einer 7%-igen Steigungs- und Gefällestrecke*, Kampf dem Larm, **20**, 4, (1973).

[27] P. E. Waters, *Commercial road vehicle noise*, J. Sound and Vibr., **35**, 153–222 (1974).

Received on 14th December 1977

ARCHIVES OF ACOUSTICS
3, 4, 249-265 (1978)

INVESTIGATIONS OF THE DIRECTIONAL PROPERTIES OF ACOUSTIC FIELD IN FINITE CUBOIDAL SPACES (GEOMETRICAL FIELD ANALYSIS)

MARIA TAJCHERT

Institute of Radio-Electronics, Technical University,
00-665 Warszawa

The paper presents an analysis of the properties of the acoustic field in a cuboidal space with the aid of the geometrical method using computer techniques. The assumptions and a brief description of the method together with the results of calculations for a space of fixed dimensions are presented. The studied properties of the acoustic field are in disagreement with results obtained by the statistical method. The directional properties of field, expressed in terms of energy, have been described as a directional characteristic of the equivalent source. Examples of plotted characteristic curves are given and the effect of various parameters on their form is discussed.

1. Introduction

Complicated physical phenomena, occurring during the propagation of acoustic waves in bounded spaces, have not yet been described satisfactorily by mathematical relations. The preferably used methods of the analysis — the wave and statistical ones — are based on the simplifying assumptions which idealize the conditions. Consequently, the results obtained from such an analysis are an idealized approach to reality. The adopted simplifying assumptions either restrict the number of cases to which the method can be applied with a sufficiently small error or result in difficult for a quantative determination errors. The assumptions of the wave method limit its application range to the regions of cuboidal or other regular form. On the other hand, the assumptions of the statistical method disregard the shape of the space, the arrangement of surfaces with different absorption coefficients, the position of the sound source and observation point, as well the decrease of the energy density due to the spherical shape of the propagating wave. These are essential simplifications since practice has shown that especially the shape of an interior and the distribution of sound absorbing materials can perceptibly affect the properties of the acoustic field. The effect of the above-mentioned factors on properties of the acoustic field is accounted for in the geometrical method of the field analysis in bounded spaces, which is based on the principles of geometrical optics. It seems that simplifying assumptions adopted in this method allow for its wider application and relatively easy estimation of errors. The application of the geometrical-graphical method is hindered by the tedious determination of image sources of higher orders and the summation of energy of individual waves reaching the observation point. This problem can be easily solved by the use of computers and only such approach to the geometrical method will enable us to take advantage of its possibilities. Straszewicz [4], dealing with the geometrical method of the field analysis, made a brief evaluation of methods used for the analysis of the acoustic field and showed the superiority of the geometrical method over the statistical and wave methods. Basing on results of the analysis of the distribution of image sources in two-dimensional areas, obtained without the use of a computer, he has come to a number of interesting conclusions. This result suggests a need for further development of the method using computers.

2. Computational techniques in geometrical method of field analysis

The method of field analysis, known as the geometrical method, is not defined uniquely. In the early days of the archictectural acoustics the geometrical method used to be defined as a method which used the laws of statistics. This method is now referred to as a statistical method. Kuttruff [2], carrying out the acoustic field analysis employing a method called by him the geometrical method, takes advantage of the mirror reflexion of the sound waves and introduces the simplifying assumptions characteristic for this method, but in the obtained relations, defining the reverberation time value, he is applying the laws of statistics. Thus his method can be termed as the geometrical and statistical method of the acoustic field analysis. The method presented below is a "pure" geometrical method of the acoustic field analysis in a bounded space, i.e. it is based exclusively on the principle of the geometrical optics. Simplifying assumptions of the method can be divided into two groups: the general — related to the very essence of the method — and the additional detailed assumptions related to a certain way of its realization.

The general assumptions for the geometrical method are the following:

1. Sound waves are replaced by "sound rays" propagating from a determined point, i.e. from a sound source. The "rays" obey the same laws of propagation as do the light rays.

2. The dimensions of the bounded space are large in comparison with the wavelength.

Additional assumptions of the presented geometrical method are:

1. The space is bounded by surfaces with a determined absorption coefficient $a \in (0, 1)$ which is independent of the angle of incidence of a "sound ray".

2. The sound source is a point source which emits a spike pulse or a very narrow noise band.

3. The change in the energy density within the investigated space results

from the propagation of a spherical wave and the absorption on the boundaries of the space.

4. Absorption of energy by the medium is neglected.

5. Diffraction and phase relationships, during propagation and reflexion, are also neglected.

6. The only interference effect of the signals reaching the observation point is the addition of the energy of waves.

As it has already been found in the previous investigations, additional simplifying assumptions are the result of the approach to the method involving the use of a simple procedure algorithm enabling the definition of certain parameters of the acoustic field. A different approach to the method and the use of another algorithm can extent the domain of applications.

Each acoustic wave reaching a given observation point, after being reflected from the surface enclosing the space, is determined uniquely by its pressure, the phase, the direction and the reverberation time. Of these four quantities only the sound pressure and the reverberation time will be described in the presented method. Below, a procedure algorithm used for this purpose is presented.

1. The determination of the position of the sound source and observation point.

2. The calculation of the distance of the image sources from a given observation point.

3. Theoretical calculations of an echogram for the purpose of determing the amplitude-time characteristic of reflections, i.e. the room impulse response

$$k(t) = \sum_n A_n \delta(t-t_n), \tag{1}$$

where A_n denotes the value of the pressure of the n-th reflection reaching the observation point after the time t_n.

If the room impulse response is determined, then it is possible to evaluate the response $s'(t)$ for a given signal $s(t)$:

$$s'(t) = \int_{-\infty}^{+\infty} s(X)k(t-X)\,dX = \sum_n A_n s(t-t_n). \tag{2}$$

4. The determination of the reflected waves pressure level L_{sc} and of the total pressure level L with the direct wave being at a given observation point

$$L_{sc} = 10\log \int_{t=0}^{\infty} s'(t)\,dt = 10\log \sum_{t=t_1}^{t_n} \Big[\sum_n A_n^2 \delta(t-t_n)\Big], \tag{3}$$

$$L_{sc} = 10\log \int_{t=t_2}^{\infty} s'(t)\,dt = 10\log \sum_{t=t_1}^{t_n} \Big[\sum_n A_n^2 \delta(t-t_n)\Big], \tag{4}$$

where t_1 is the arrival time of the direct wave and t_2 is the arrival time of the first reflected wave.

3. Subject of analysis

In this paper the properties of the acoustic field in a cuboidal space will be analyzed with the aid of the geometrical method using computers.

Since the laws of the propagation of a wave propagating directly from the sound source are precisely defined and can be introduced at any moment into the analysis, the main emphasis will be placed on the properties of the acoustic field of the reflected waves [4]. The main parameter defining the field in the method used is the spatial distribution of the reflected waves pressure. An analysis of this distribution will constitute an essential part of this paper.

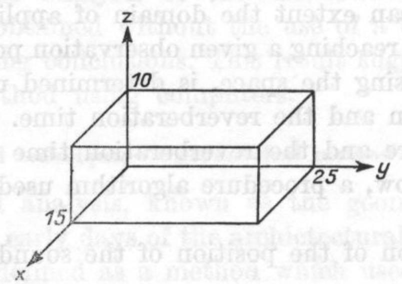


Fig. 1. The position of the tested enclosed space in the coordinate system

The limitation of the range of analysis to only one form of the cuboidal space has resulted in such advantages as clear form of the results and the possibility of systematic investigation of the changes of the field parameters as functions of other parameters such as the distance between the source and observation point, their arrangement within the space, and also the absorption coefficient at various surfaces. In the cuboidal space the distribution of the image sources and its changes under the influence of the above-mentioned parameters are easy to predict and, since this distribution determines the properties of the acoustic field, one can discuss without difficulty the results obtained by computation. Furthermore, the use of the geometrical method for the cuboidal spaces has enabled to evolve a simple and concise computation program, e.g. in comparison with a program for the region of any shape [6].

In the examples of the use of computers in the geometrical method of the field analysis, as cited in the literature [5, 7], preference is given to the "ray method" while the forms of the analyzed spaces are rather simple for reasons mentioned before. The properties of the acoustic field – found for the cuboidal spaces – can, to some approximation, be extended to the acoustic field in regions of different form, but the distribution of the apparent sources in a given case should be accounted for, at least qualitatively. Obviously, for the accurate calculations, a suitably elaborated program is a necessity.

4. Outline of the program

The program has been written in the FORTRAN language; the calculations were performed on CYBER CDC 6000 computer. Below the block diagram of the program is given.

Data

↓

Calculation of one coordinate determining the distance from the observation point as well as the number of reflections from either of two walls perpendicular to the axis on which the coordinate is calculated

↓

Calculation of the distance of the sound source from the observation point, the approach time of the reflected wave to the observation point, the losses arising from reflection as well as pressure of subsequent reflected waves, i.e. of echoes

↓

Arrangement of results in sequence of the growth arrival time of echoes

↓

Addition of the energies of echoes, the difference of the arrival times being C

↓

Calculation of the total pressure level L as well as of the pressure level of the sum of reflected waves L_{sc}

↓

output of the result

The input data of the program are: dimensions of the bounded space, coordinates of the sound source and observation point, absorption coefficients of individual surfaces, the time interval C over which energy of reflection is summed, pressure level of the direct wave at a distance of 1 m from the sound sorce, the order of reflections $\leqslant 10$ to be considered. The results of calculations are: a full theoretical echogram containing all the echoes and giving the sequence of reflections from particular walls, an abbreviated echogram, i.e. the echogram obtained after the summing up the energies of the reflected waves which reach the observation point in sufficiently small time intervals, the reflected waves pressure level L_{sc} and the total pressure level L at the observation point.

5. The results of investigations and their interpretation

The calculations have been performed for a cuboidal space of dimensions $15 \times 25 \times 10$ m, with a point sound source radiating the wave with a pressure level 100 dB at a distance of 1 m.

To limit the computation, time reflections up to the 7-th order were taken into account and this seems to be sufficient considering that the last reflected wave reaching the observation point after approximately 500 ms has a pressure level lower by 35-40 dB than the first one. The time interval C is equal to 0,1 ms and this practically ensures the summation of the waves reaching the observation point on the paths of equal length. The position of the sound source and observation point and the surface absorption coefficient are changed in the calculations.

The computations yield interesting results on essential spatial properties of the acoustic field which do not agree with the results of the analysis performed by statistical method, namely:

1. variability of the reflected waves pressure level L_{sc} as a function of the distance from the source r;

2. relation between the reflected waves pressure level L_{sc} and the position of the sound source and observation point in the space at a constant distance r from the source;

3. relations between the reflected waves pressure level L_{sc} and the arrangement of boundary surfaces with different absorption coefficients.

The variability of the reflected waves pressure level as a function of the distance of the observation point from the sound source was observed by Straszewicz [1] in the case of non-rectangular shapes of the projections of the interiors. For this purpose he took advantage of the geometrical analysis method without resort to computation. The results of calculations of the reflected waves pressure level L_{sc} obtained with the aid of the geometrical method, as shown in Figs. 1–5, are compared with those obtained by the statistical method [1].

The comparison of these results is complicated because of the fact that in the geometrical method the value of the absorption coefficient α_c should be close to the value of the physical coefficient while in the statistical method — to the value of the reverberation coefficient α_p. The relation between these two coefficients [3] depends on the frequency and absolute value of coefficients. In all comparative calculations performed by the statistical method it has been assumed that

$$\frac{\alpha_p}{\alpha_c} = \begin{cases} 1.5 & \text{for } \alpha_c \leqslant 0.5, \\ 1.2 & \text{for } 0.5 \leqslant \alpha_c \leqslant 0{,}8, \\ 1 & \text{for } \alpha_c > 0.8. \end{cases} \tag{5}$$

The analysis of individual groups of results leads to the conclusions which are summarized below. The reflected waves pressure level L_{sc} is plotted in Fig. 2 as a function of the distance from the source r, for several positions of the source and the pressure level L_{sc} calculated by the statistical method, with the absorption coefficient level being equal at all the surfaces (Fig 2a — $\alpha = 0.1$; Fig. 2b — $\alpha = 0.6$).

Remark 1. The reflected waves pressure level decreases with the growth of the distance from the source.

Remark 2. The rate of changes of the pressure level L_{sc} depends on the position of the sound source; these changes are the faster the closer lies the

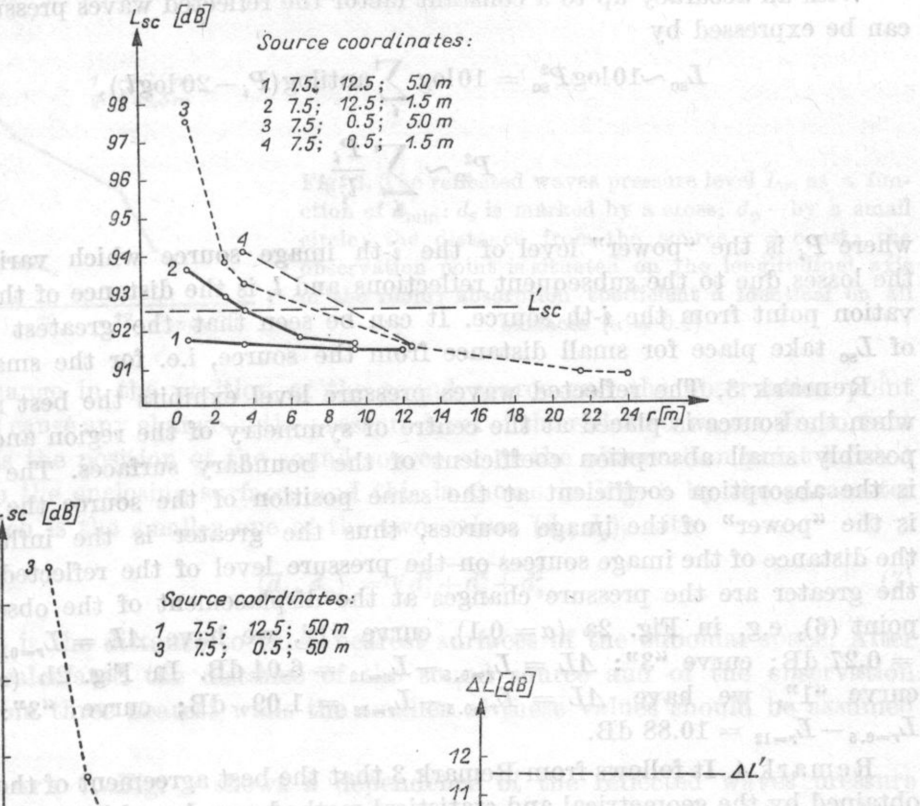


Fig. 2. The reflected waves pressure level L_{sc} as a function of the distance from the source r

The observation point is located on the longitudinal axis of the room; absorption coefficients identical on all surfaces: (a) $\alpha = 0.1$; (b) $\alpha = 0.6$; (c) difference between reflected pressure level at $\alpha = 0.1$ and $\alpha = 0.6$ for two positions of the source (the coordinates as shown in Fig. 1a). For comparison the values calculated by the statistical method are given

sound source or observation point to the boundaries of the space; in a given position of the source the greatest differences in the pressure level of the reflected waves occur for small distances, e.g., in Fig. 2a curve "1" when the source lies in the centre of symmetry of the space $\Delta L = L_{r=0.5} - L_{r=3.5} = 0.07$ dB, and $\Delta L = L_{r=3.5} - L_{r=12} = 0.19$ dB; curve "3", when the source is close to the wall $\Delta L = L_{r=0.5} - L_{r=3.5} = 4.34$ dB, $\Delta L = L_{r=3.5} - L_{r=12} = 1.7$ dB.

With an accuracy up to a constant factor the reflected waves pressure level can be expressed by

$$L_{sc} \sim 10 \log P_{sc}^2 = 10 \log \sum_i \operatorname{antilog}(P_i - 20 \log l_i), \tag{6}$$

$$P_{sc}^2 \sim \sum_i \frac{P_i}{l_i^2}, \tag{6a}$$

where P_i is the "power" level of the i-th image source which varies with the losses due to the subsequent reflections and l_i is the distance of the observation point from the i-th source. It can be seen that the greatest changes of L_{sc} take place for small distance from the source, i.e. for the smallest l_i.

Remark 3. The reflected waves pressure level exhibits the best stability when the source is placed at the centre of symmetry of the region and at the possibly small absorption coefficient of the boundary surfaces. The greater is the absorption coefficient at the same position of the source the smaller is the "power" of the image sources, thus the greater is the influence of the distance of the image sources on the pressure level of the reflected waves, the greater are the pressure changes at the displacement of the observation point (6), e.g. in Fig. 2a ($\alpha = 0.1$), curve "1", we have $\Delta L = L_{r=0.5} - L_{r=12} = 0{,}27$ dB; curve "3": $\Delta L = L_{r=0.5} - L_{r=12} = 6.04$ dB. In Fig. 2b ($\alpha = 0.6$), curve "1", we have $\Delta L = L_{r=0.5} - L_{r=12} = 1.09$ dB; curve "3": $\Delta L = L_{r=0.5} - L_{r=12} = 10.88$ dB.

Remark 4. It follows from Remark 3 that the best agreement of the results obtained by the geometrical and statistical methods can be achieved by placing the sound source close to the centre of symmetry of the space. The regularity of the arrangement of the apparent sources brings about that the displacement of the observation point leads to only minor changes of the value of L_{sc}.

Remark 5. The increase of absorption coefficients on the boundary surfaces causes faster changes of L_{sc} as a function of the distance r (for explanation see item 3). Fig. 2c shows the value of the difference of levels ΔL of the reflected waves as a function of the distance from the source, with absorption coefficients $\alpha = 0.1$ and $\alpha = 0.6$ for two positions of the source. It can be seen that the rate of changes of ΔL decreases when the source is moving away, with their increasing absolute value, i.e., the increase of the absorption coefficient causes the greater changes the greater is their distance from the source.

Remark 6. With various positions of the source and the constant distance of the observation point, with identical absorption coefficients on the surfaces, the reflected waves pressure level is the smaller the greater is the distance of the sound source or observation point from the boundary surface. Considering the symmetry of the spatial structure of the network of apparent sources for a cuboidal space, it is possible to interchange a position of the sound source and the observation point.

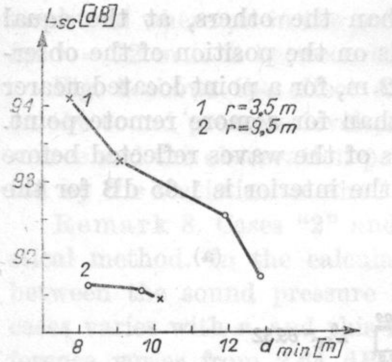


Fig. 3. The reflected waves pressure level L_{sc} as a function of $d_{\min}$: d_s is marked by a cross; d_p – by a small circle; the distance from the source r = const; the observation point is situated on the longitudinal axis of the room; absorption coefficient α identical on all surfaces ($\alpha = 0.1$)

A change in the position of the sound source and the observation point does not cause any change in the pressure level of the reflected waves. Important for L_{sc} is the position of the sound source or of the observation point located closer to the enclosing surfaces and this is shown in Fig. 3 by the parameter $d_{\min}$ which is the smaller one of the two values (d_s, d_p), with

$$(d_s, d_p) = \sqrt{d_1^2 + d_2^2 + d_3^2}, \tag{7}$$

where d_i is the distance to three nearest surfaces of the cuboidal space. After having calculated the distance of the sound source and of the observation point from three nearest walls the smaller of these values should be assumed as $d_{\min}$.

Remark 7. Fig. 4 shows a dependence of the reflected waves pressure level on the distribution of sound absorbing materials. If one of the surfaces

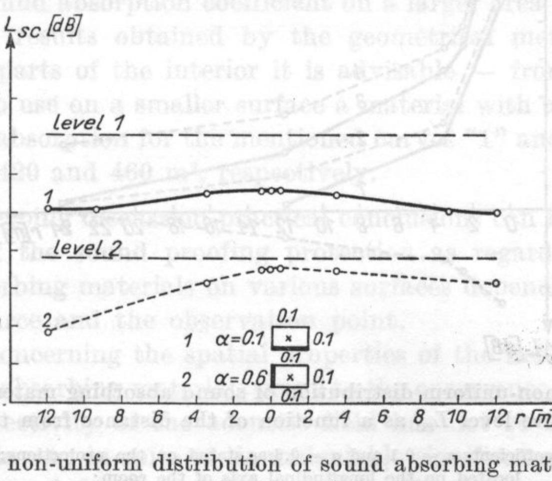


Fig. 4. The effect of non-uniform distribution of sound absorbing materials on the reflected waves pressure level L_{sc} as a function of the distance from the source r

Absorption coefficient of one surface much higher than that of the others ($\alpha = 0.6$, the others $\alpha = 0.1$ as stated on the projection); the observation point is situated on the longitudinal axis of the room; the coordinates of the source 7.5; 12.5; 5.0 m. For comparison $L_{sc}(r)$ has been plotted with $\alpha = 0.1$ at all surfaces and L'_{sc} for bot cases has the sound absorption coefficient greater than the others, at the equal distance from the source, the value of L_{sc} depends on the position of the observation point relative to this surface : when $r = 12$ m, for a point located nearer the absorbing surface L_{sc} is smaller by 0.6 dB than for a more remote point. The difference between the sound pressures levels of the waves reflected before and after the change in the absorbing power of the interior is 1.65 dB for the

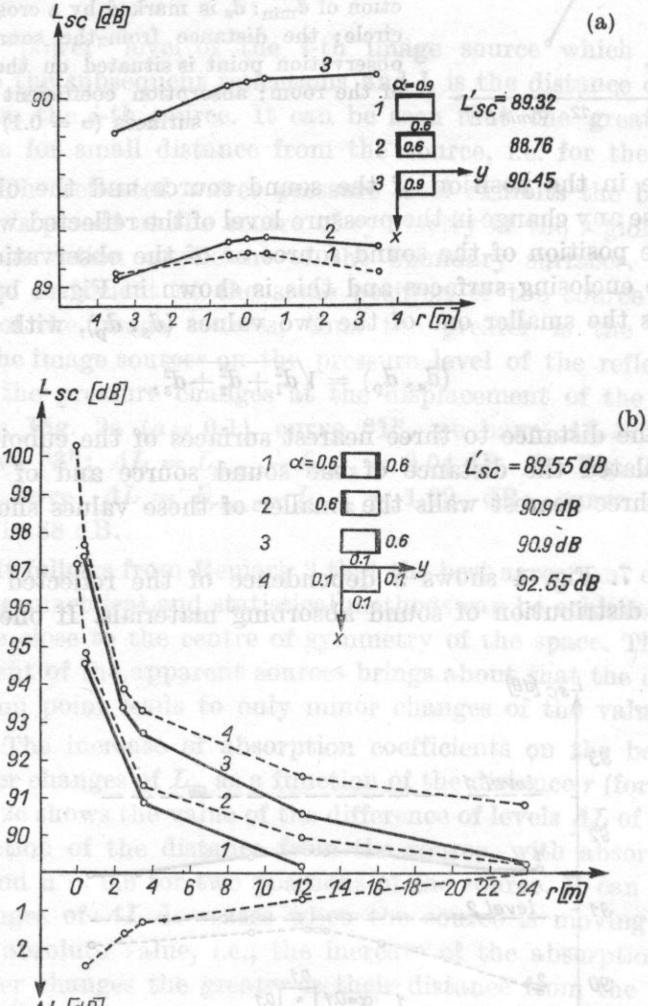


Fig. 5. The effect of non-uniform distribution of sound absorbing materials on the reflected waves pressure level L_{sc} as a function of the distance from the source r

Surface absorption coefficients $\alpha = 0.1$ and $\alpha = 0.6$ as stated on the projections; the observation point is located on the longitudinal axis of the room:

(a) the coordinates of the source 7.5; 12.5; 5.0 m.

(b) the coordinates of the source 7.5; 0.5; 5.0 m; in the bottom portion of the graph the course of $\Delta L(r)$ has been plotted, where $\Delta L = L_{sc3} - L_{sc2}$;

The values of L_{sc} calculated by the statistical method for particular case are given statistical where as in the geometrical method, it differs for various directions: for $r = 12$ m it is between 0.96 and 1.61 dB.

Fig. 5 shows the effect of the arrangement of sound absorbing materials on the sound pressure level of the reflected waves at the same position of the source and the observation point. For comparison the values of L_{sc} are calculated by the statistical method.

Remark 8. Cases "2" and "3" are equivalent in calculations by the statistical method. In the calculations by the geometrical method the difference between the sound pressure levels of the reflected waves ΔL for these two cases varies with r, and this is shown in the bottom part of Fig. 5a: this difference varies from 2.64 dB for $r = 0.5$ m to 0.65 dB for $r = 12$ m.

Remark 9. If the source and the observation point are at equal distance from opposite walls, then the use of sound absorbing materials on each of either walls gives the same result in the case of consideration of L_{sc}, e.g., in Fig. 5a curves "2" and "3" for $r = 24$ m, $L_{sc} = \text{const.}$

Remark 10. For non-uniform distribution of sound absorbing materials the sound pressure level of the reflected waves may not be the highest at the source point as is the case of the uniform distribution (Fig. 3, curve "1"). Although for the source point the sum of distances from the apparent sources is the smallest, their different "power", being determined by the distribution of sound absorbing materials, causes the displacement of the point having a maximum sound pressure level of the reflected waves towards the surface with a smaller absorbtion coefficient (Fig. 5b).

Remark 11. Using the statistical method for curves "1" and "2", it can be seen from Fig. 5b that a lower level of L_{sc} can be obtained by using a material with a smaller sound absorption coefficient on a larger area ("2"). On the other hand, basing on results obtained by the geometrical method, it is evident that for certain parts of the interior it is advisable — from the viewpoint of reducing L_{sc} — to use on a smaller surface a material with a greater coefficient ("1"). The total absorption for the mentioned curves "1" and "2" is comparable and amounts to 420 and 460 m², respectively.

From the foregoing discussion practical conclusions can be drawn, e.g. from the viewpoint of the sound proofing protection as regards the efficiency of using sound absorbing materials on various surfaces depending on the position of the sound source and the observation point.

All remarks concerning the spatial properties of the field for various distributions of sound absorbing materials point to the occurrence of some directional features. The directivity of the acoustic field has not so far been defined in an inobjectionable way in spite of its undeniable significance for the perception of sounds and the evaluation of the interiors. In the method used the field at a given observation point is defined by the pressure level of the reflected waves, i. e. by a quantity proportional to the energy density. Therefore, an approach in terms of power to the directivity of the acoustic field has been proposed. Taking into account the distribution of energy in the field of the reflected waves, the action of an omnidirectional sound source in an enclosed space can be replaced by the action of a source with a determined direction characteristics in free space. In other words, the effect of the system geometry (the space enclosed by the surfaces with determined absorption coefficients, the omnidirectional source, the observation point) on the distribution of energy in the field of the reflected waves becomes transformed into the directional characteristic of the sound source radiating in free space. Such approach does not provide the information about the direction of the energy reaching a given observation point, but only about mutual relations of the energy of the wave reaching various points of the field.

If we have a cuboidal space in which the position of the sound source and of the observation point are defined and if we have a spatial polar coordinates system (r, φ, θ), whose centre lies in the source point, and thus the coordinates of the source are $(0, 0, 0)$, the coordinates of the observation point (r, φ, θ) and the coordinates of the reference point $(r_0, \varphi_0, \theta_0)$, then the pressure at the observation point is p_φ, at the reference point — p_0. The power directivity of the field is defined by the directivity coefficient of the equivalent source q,

$$q(r, \varphi, \theta) = \frac{P_\varphi}{P_0}, \tag{8}$$

where P_φ denotes the power of the omnidirectional source which at the observation point in a free space produces the pressure p_φ, P_0 is the power of the omnidirectional source which at the reference point in free space produces the pressure p_0. By determining two of the three coordinates of the observation point one can obtain suitable directional characteristics of the equivalent source,

$$q(\varphi)_{\substack{r=\mathrm{const}\\ \theta=\mathrm{const}}} = \frac{p^2(r, \varphi, \theta)}{p^2(r, 0, \theta)}, \quad q(\theta)_{\substack{r=\mathrm{const}\\ \varphi=\mathrm{const}}} = \frac{p^2(r, \varphi, \theta)}{p^2(r, \varphi, 0)},$$

$$q(r)_{\substack{\varphi=\mathrm{const}\\ \theta=\mathrm{const}}} = \frac{p^2(r, \varphi, \theta)}{p^2(0, 0, 0)}, \tag{9}$$

where p is the value of the sound pressure of the sum of the reflected waves at a point with given coordinates. If for a given observation point (r, φ, θ) the directivity index is greater unity, e.g. $q(\varphi) > 1$, this means that in the direction φ more energy is concentrated than in the direction $(r, 0, \theta)$. To put it the other way, in order to obtain such a pressure level as it is at the point (r, φ, θ) of the investigated field, it is necessary to use the source with a q times greater power than the power of the source permitting to obtain a determined pressure level at the point $(r, 0, \theta)$ in the case for which the source would function in free space. With an arbitrary choice of parameters and of a variable out of (r, φ, θ),

$q > 1$ means a gain of energy. Figs. 6–8 show – in terms of logarithmic measures – the determined directional characteristic of the equivalent source, that is, a directional gain $\boldsymbol{q}$,

$$\boldsymbol{q}(r, \varphi, \theta) = 10 \log \frac{P_\varphi}{P_0} = 10 \log q(r, \varphi, \theta) \, \mathrm{dB}, \tag{10}$$

that is to say, the directional gain is indicated by $\boldsymbol{q} > 0$ dB. The omnidirectional characteristic is represented by a circle with the power 0 dB; at points located inside the circle a loss of energy ($\boldsymbol{q} < 0$ dB) occurs in relation to the reference point, at points outside the circle there is a gain of energy ($\boldsymbol{q} > 0$ dB). The determined points of the characteristics connected by the straight lines do not show the shape of the characteristic, but merely some tendencies and regularities. The graphical presentation of more accurate characteristic would require the determination of a considerably greater number of points than four, nevertheless within the cuboidal space under consideration one might not expect larger irregularities (even with the determined four points the interpretation of the characteristic would be possible). To facilitate the interpretation of the characteristic, three groups of cases have been established:

1. the sound source is located at the centre of symmetry of the space, different absorption coefficients of boundary surfaces;
2. the sound source is located at any point of the space, equal absorption coefficients of boundary surfaces;
3. the sound source is located at any point of the space, different absorption coefficients of boundary surfaces.

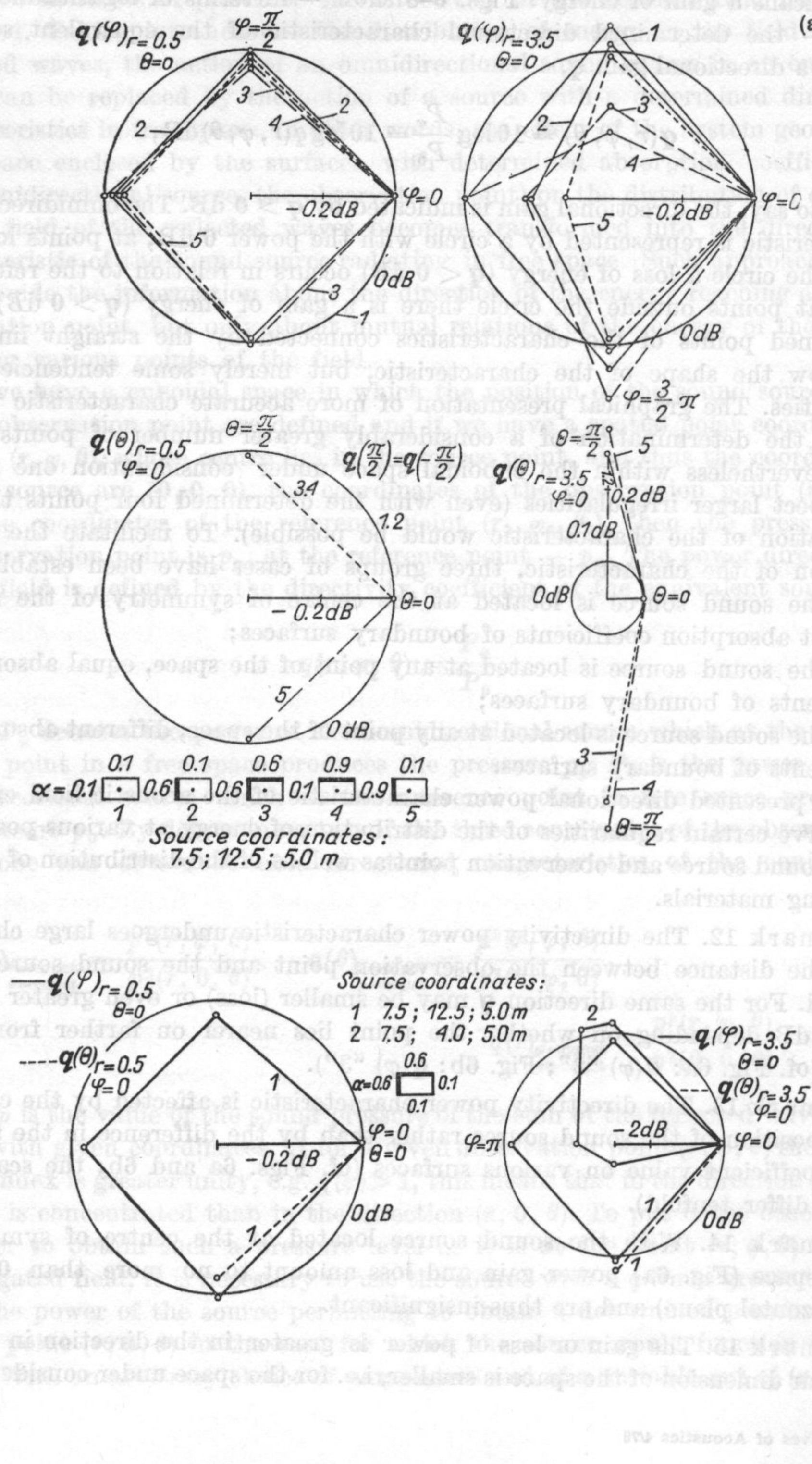
The presented directional power characteristic of the acoustic field enables to observe certain regularities of the distribution of energy at various positions of the sound source and observation point as well as of the distribution of sound absorbing materials.

Remark 12. The directivity power characteristic undergoes large changes when the distance between the observation point and the sound source r is changed. For the same direction $\boldsymbol{q}$ may be smaller (loss) or even greater (gain) than 0 dB depending on whether the point lies nearer on farther from the source (of. Fig. 6a: $\boldsymbol{q}(\varphi)$ "2"; Fig. 6b: $\boldsymbol{q}(\varphi)$ "3").

Remark 13. The directivity power characteristic is affected by the change in the position of the sound source rather than by the difference in the absorption coefficient value on various surfaces (cf. Figs. 6a and 6b; the scales of figures differ tenfold).

Remark 14. With the sound source located at the centre of symmetry of the space (Fig. 6a) power gain and loss amount to no more than 0.2 dB (in horizontal plane) and are thus insignificant.

Remark 15. The gain or loss of power is greater in the direction in which the linear dimension of the space is smaller, i.e. for the space under consideration


(a)
$q(\varphi)_{r=0.5}$
$\theta=0$
$\varphi=\frac{\pi}{2}$
$\varphi=0$
$-0.2dB$
$0dB$
$q(\varphi)_{r=3.5}$
$\theta=0$
$\varphi=\frac{3}{2}\pi$
$q(\theta)_{r=0.5}$
$\varphi=0$
$\theta=\frac{\pi}{2}$
$q(\frac{\pi}{2})=q(-\frac{\pi}{2})$
$q(\theta)_{r=3.5}$
0.2dB
0.1dB
$\theta=0$
$\alpha=0.1$
Source coordinates:
7.5; 12.5; 5.0 m
$q(\varphi)_{r=0.5}$
$q(\theta)_{r=0.5}$
Source coordinates:
1 7.5; 12.5; 5.0 m
2 7.5; 4.0; 5.0 m
$\alpha=0.6$
$q(\varphi)_{r=3.5}$
$q(\theta)_{r=3.5}$
$\varphi=\pi$
$-2dB$

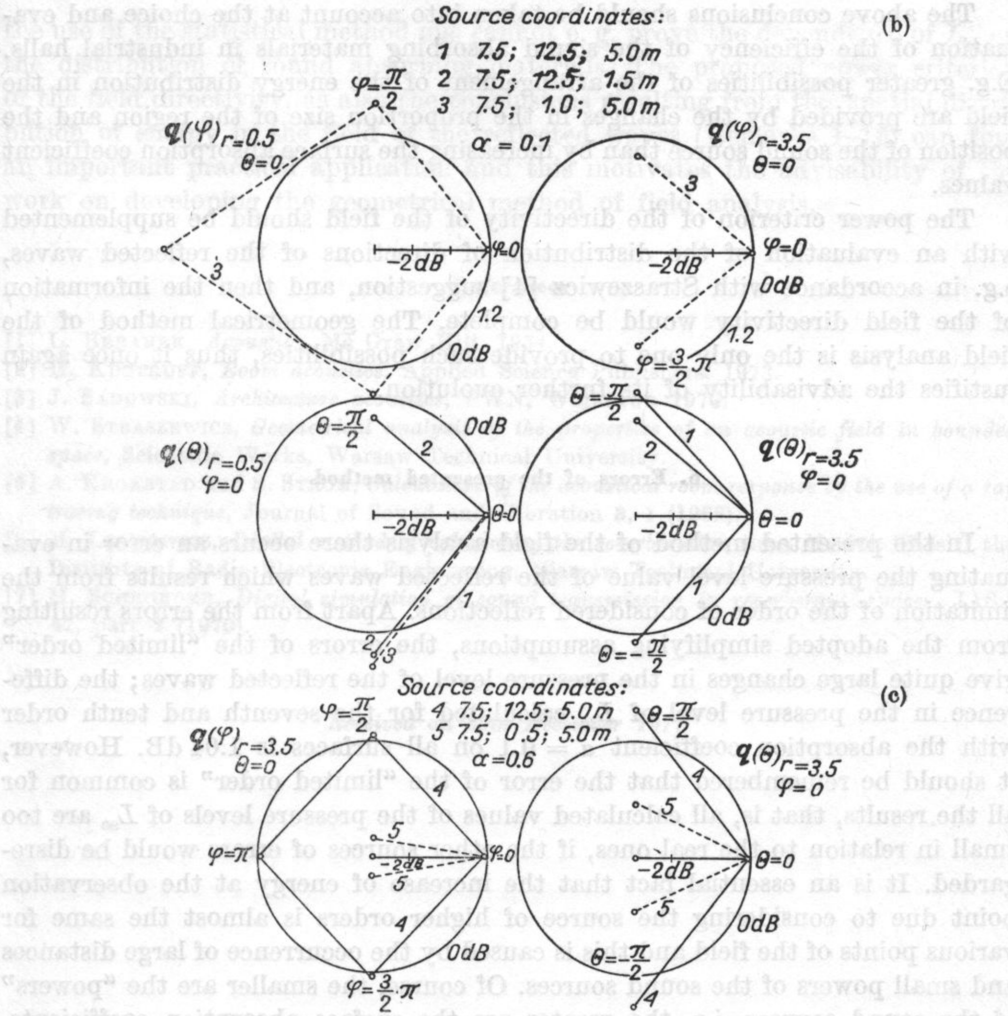


Fig. 6. The directional characteristics, expressed in terms of logarithmic values of the equivalent source, defining the directional power properties of the field

(a) The source situated at the centre of symmetry of the space different surface absorption coefficients as stated on projections
(b) The sound source situated at any point of the space enclosing surfaces with equal absorption coefficients $\alpha=0.1$ and $\alpha=0.6$.
(c) The sound source situated at any point of the space enclosing surfaces with different absorption coefficients according to the sketch on the projection

greater changes can be observed on the characteristic $\boldsymbol{q}(\theta)$ than on the characteristic $\boldsymbol{q}(\varphi)$, especially for the source located at the centre of symmetry (Fig. 6a, $\boldsymbol{q}(\varphi)$ and $\boldsymbol{q}(\theta)$).

Remark 16. The change in the directivity power characteristics is the greater the greater is the surface absorption coefficient (Fig. 6b, "1" and "4") as well as the greater are the differences between the coefficient values (Fig. 6a, "2" and "4").

The above conclusions should be taken into account at the choice and evaluation of the efficiency of the sound absorbing materials in industrial halls. E.g. greater possibilities of the arrangement of the energy distribution in the field are provided by the changes in the proportion size of the region and the position of the sound source than by increasing the surface absorption coefficient values.

The power criterion of the directivity of the field should be supplemented with an evaluation of the distribution of directions of the reflected waves, e.g. in accordance with Straszewicz [4] suggestion, and then the information of the field directivity would be complete. The geometrical method of the field analysis is the only one to provide such possibilities, thus it once again justifies the advisability of its further evolution.

6. Errors of the presented method

In the presented method of the field analysis there occurs an error in evaluating the pressure level value of the reflected waves which results from the limitation of the order of considered reflections. Apart from the errors resulting from the adopted simplifying assumptions, the errors of the "limited order" give quite large changes in the pressure level of the reflected waves; the difference in the pressure level of L_{sc} calculated for the seventh and tenth order with the absorption coefficient $a = 0.1$ on all surfaces is 1.04 dB. However, it should be remembered that the error of the "limited order" is common for all the results, that is, all calculated values of the pressure levels of L_{sc} are too small in relation to the real ones, if the other sources of errors would be disregarded. It is an essential fact that the increase of energy at the observation point due to considering the source of higher orders is almost the same for various points of the field and this is caused by the occurrence of large distances and small powers of the sound sources. Of course, the smaller are the "powers" of the sound sources, i.e. the greater are the surface absorption coefficients, the smaller are the errors of the "limited order". For the calculated cases, for which the absorption coefficients are small ($a = 0.1$, $a = 0.6$), these errors are large, but considering their regularity the conclusions drawn from the calculated spatial distribution of the pressure levels of L_{sc} can be considered to be right.

7. Conclusions

The use of computers in the geometrical analysis of acoustic field has enabled us to find the properties of the field which differ from the ones obtained by the statistical method, e. g. the variability of the pressure level of the reflected waves as a function of the distance from the source; also with the use of the statistical method one cannot e. g. prove the dependence of L_{sc} on the distribution of sound absorbing materials. The proposed power criterion of the field directivity, as also the conclusions resulting from the spatial distribution of energy in the field of the reflected waves (Remarks 1–16) can find an important practical application and this motivates the advisability of the work on developing the geometrical method of field analysis.

References

[1] L. Beranek, *Acoustics*, Mc Graw Hill, 1954.
[2] H. Kuttruff, *Room acoustics*, Applied Science Publishers, 1973.
[3] J. Sadowski, *Architecture acoustics*, PWN, Warszawa 1976.
[4] W. Straszewicz, *Geometrical analysis of the properties of an acoustic field in bounded space*, Scientific Works, Warsaw Technical University.
[5] A. Krokstad and S. Strom, *Calculation of the acoustical room response by the use of a ray tracing technique*, Journal of Sound and Vibration 8, 1 (1968).
[6] M. Leonowicz, *Digital method of calculating the reverberation time*, M. Sci. Thesis, the Institute of Radio-Electronic Engineering, Warsaw Technical University.
[7] M. Schroeder, *Digital simulation of sound transmission in reverberant spaces*, JASA 47, part 2 (1970).

Received on 20th December 1977

ARCHIVES OF ACOUSTICS
3, 4, 267-271 (1978)

TEMPERATURE AND PRESSURE CHANGES OF THE "COLLISION FACTOR" IN SCHAAFFS' MOLECULAR-KINETIC THEORY OF WAVE PROPAGATION IN LIQUIDS

EUGENIUSZ SOCZKIEWICZ

Institute of Physics, Silesian Technical University,
44-100 Gliwice, Krzywoustego 2

Expressions have been derived which describe the pressure coefficient and temperature coefficient at constant pressure and volume of the "collision factor" s which determines, in Schaaffs' molecular-kinetic theory, the values of the velocity and attenuation coefficient of ultrasonic waves propagating in liquid. The values of the former coefficients have been determined for homologous series of saturated hydrocarbons, alkyl iodides, and benzene derivatives.

1. Introduction

According to Schaaffs' molecular–kinetic theory [4], p. 253, the velocity of ultrasonic waves in liquid is determined by the space filling and the elasticity of molecular collisions and is expressed by the formula

$$w = w_{\infty} s r, \tag{1}$$

where w is the propagation velocity of ultrasonic waves, w_{∞} – a constant coefficient equal to 1600 [m/s], s – collision factor (*Stossfaktor*) determining the elasticity of the collisions of the molecules of the liquid, $r = B/V$, B being the specific volume of a mol of molecules, and V – molar volume of the liquid.

The value of the collision factor s determines also the attenuation of ultrasonic waves in liquid. According to Schaaffs ([4], p.437) the attenuation coefficient of ultrasonic waves in non-relaxation region is expressed as

$$\frac{\alpha}{\nu^2} = C\frac{4-s}{s^4 r^3}, \tag{2}$$

where α – attenuation coefficient, ν – wave frequency, C – constant coefficient determined by Schaaffs and equal to 1.1×10^{-13} [s²/m]. Thus the temperature and pressure dependencies of the collision factor define the corresponding relations for the attenuation coefficient of ultrasonic waves.

2. Temperature dependence of the collision factor

Schaaffs, Kuhnkies and Woelk [5] have demonstrated that the temperature dependence of the collision factor in some liquids can be expressed by

$$s = 4\left(1-\frac{T}{962}\right), \tag{3}$$

where T is the absolute temperature.

Sette [6] (cf. also [1], p. 247) has shown, using Rao's rule expressed in the form

$$w^{1/3}V = \text{const.} \tag{4}$$

and the Sugden formula for the dependence of the density of liquid on temperature

$$\varrho-\bar{\varrho} = \varrho_0\left(1-\frac{T}{T_c}\right)^{0,3}, \tag{5}$$

where ϱ is the density of the liquid, $\bar{\varrho}$ — the density of saturated vapour, ϱ_0 — the density at absolute zero temperature, T_c — the critical temperature, that

$$s = K\left(1-\frac{T}{T_0}\right)^{0.6}, \tag{6}$$

where K is a constant characteristic for the given liquid and depends, among other, on ϱ_0.

In the present paper, temperature coefficients of the collision factor at constant pressure and volume have been determined using the generalized Rao's rule ([4], p. 281)

$$w^{6/n}\,V = \text{const.}, \tag{7}$$

where the exponent $6/n$ is individual for each liquid, and from the Kuczera [2] formula for temperature coefficient of ultrasound velocity at constant volume

$$\frac{1}{w}\left(\frac{\partial w}{\partial T}\right)_V = \frac{7}{6}\gamma, \tag{8}$$

where γ is the bulk coefficient of expansion of liquid at constant pressure. It follows from (1) and (7), after simple calculations, that

$$\frac{1}{s}\left(\frac{\partial s}{\partial T}\right)_p = \left(1-\frac{n}{6}\right)\gamma, \tag{9}$$

and from (1) and (8)

$$\frac{1}{s}\left(\frac{\partial s}{\partial T}\right)_V = \frac{7}{6}\gamma. \tag{10}$$

Table 1 summarizes temperature coefficients $(1/s)(\partial s/\partial T)_p$ and $(1/s)(\partial s/\partial T)_V$ of the collision factor calculated from the above formulae for homologous series of saturated hydrocarbons, alkyl iodides and benzen derivatives. The exponents $6/n$ have been determined from the relation implied by Rao's rule (7)

$$\frac{1}{w}\left(\frac{\partial w}{\partial T}\right)_p = \frac{n}{6}\gamma, \tag{11}$$

and using the data from Landolt-Börnstein Tables [3] and the data of Bergmann ([1], p. 235).

Table 1. The values of temperature coefficients of the collision factor $(1/s)(\partial s/\partial T)_p$ and $(1/s)(\partial s/\partial T)_V$ determined from equations (9) and (10)

Substance	$\gamma \times 10^5\,[\mathrm{K}^{-1}]$	n	$-\frac{1}{s}\left(\frac{\partial s}{\partial T}\right)_p \times 10^5$	$\frac{1}{s}\left(\frac{s}{T}\right)_V \times 10^5$
n-pentane	161.00	19.20	354.20	187.80
n-hexane	135.00	16.86	244.30	157.50
n-heptane	124.40	17.30	234.30	145.10
n-octane	114.00	17.36	215.90	133.00
n-nonane	102.00	18.11	205.90	119.00
n-decane	101.50	17.43	193.30	118.40
n-dodecane	96.20	17.77	188.90	112.20
n-tetradecane	89.40	18.58	187.40	104.30
n-hexadecane	80.00	19.32	177.70	93.80
methyl iodide	125.00	17.28	234.90	145.80
ethyl iodide	116.90	17.10	216.80	136.40
propyl iodide	109.50	17.11	202.70	127.70
butyl iodide	102.00	15.65	164.10	119.00
benzene	123.00	19.12	269.00	143.50
fluorobenzene	116.00	18.14	234.70	135.30
chlorobenzene	98.00	17.57	188.90	114.30
bromobenzene	90.30	17.72	176.40	105.30
iodobenzene	83.00	17.40	157.70	96.80

3. Pressure dependence of the collision factor

It follows from thermodynamic considerations that the temperature coefficient of the ultrasound velocity at a constant pressure can be written in the form

$$\left(\frac{\partial w}{\partial T}\right)_p = \frac{\partial(w,p)}{\partial(T,p)} = \frac{\partial(w,p)}{\partial(T,V)}\,\frac{\partial(T,V)}{\partial(T,p)}, \tag{12}$$

where $\partial(w,p)/\partial(T,p)$, $\partial(w,p)/\partial(T,V)$, and $\partial(T,V)/\partial(T,p)$ are the respective jacobians.

By specifing relation (12) one obtains

$$\left(\frac{\partial w}{\partial T}\right)_p = \left(\frac{\partial w}{\partial T}\right)_V - \left(\frac{\partial w}{\partial p}\right)_T \cdot \frac{\gamma}{\beta_T}, \tag{13}$$

since $(\partial p/\partial T)_V = \gamma/\beta_T$, where β_T is the isothermal coefficient of the compressibility of liquid. By combining equations (1), (8), (11), and (13) one obtains an expression for the pressure coefficient of the collision factor:

$$\frac{1}{s}\left(\frac{\partial s}{\partial p}\right)_T = \frac{n+1}{6}\beta_T. \tag{14}$$

Table 2 presents the values of coefficients $(1/s)(\partial s/\partial p)_T$ calculated from equation (14). The values β_T refer to normal pressure and have been calculated from the formula

$$\beta_T = \frac{\varkappa}{\varrho w^2}, \tag{15}$$

where $\varkappa$ is the ratio of the specific heats at constant pressure and constant volume, calculated from (1) and (7),

$$\varkappa = 1 + \frac{T\gamma^2 w^2}{c_p}, \tag{16}$$

where c_p is the specific heat at constant pressure per unit mass of the liquid.

Table 2. The values of coefficients $(1/s)(\partial s/\partial p)_T$ at 293 K from equation (14)

Substance	$\beta_T \times 10^{11}$ [m²/N]	$\frac{1}{s}\left(\frac{\partial s}{\partial p}\right)_T \times 10^{10}$ [m²/N]
n-pentane	210.80	71.00
n-hexane	157.17	54.60
n-heptane	142.61	43.50
n-octane	124.10	38.00
n-nonane	112.50	35.80
n-decane	115.52	35.50
n-dodecane	95.46	29.90
n-tetradecane	88.04	28.70
n-hexadecane	84.77	28.70
benzene	94.86	31.80
fluorobenzene	94.54	30.20
chlorobenzene	80.44	24.90
bromobenzene	65.48	20.40
iodobenzene	59.51	18.30

4. Conclusions

It follows from an analysis of Tables 1 and 2 that:

(a) The collision factor decreases with temperature increase if heating is carried at a constant pressure since the derivative $(\partial s/\partial T)_p$ is negative. In the homologous series the values of $(1/s)(\partial s/\partial T)_p$ decrease as the number of the homologue grows.

(b) The collision factor increases with temperature increase if heating is carried at a constant volume since the derivative $(\partial s/\partial T)_V$ is positive. In the homologous series the values of $(1/s)(\partial s/\delta T)_V$ decrease as the number of the homologue grows.

(c) The collision factor grows with pressure increase and the values of coefficients in homologous series decrease as the number of the homologue grows.

Schaaffs formula (2) as well as relations (9) and (14) derived in this paper make it possible to calculate temperature and pressure coefficients of ultrasonic wave attenuation. In the case of benzene derivatives the calculated coefficients are only in qualitive agreement with experimental data. This problem was treated in detail elsewhere [8].

References

[1] L. Bergmann, *Ultrazvuk i jevo primenenije w naukie i technikie*, Izdat. Inostrannoj Literatury, Moskwa 1957, 214.

[2] F. Kuczera, *Termiczeskije svojstva skorosti ultrazvuka v židkostjach postojannoj plotnosti*, Primenenije ultraakustiki k issledovanii veszczestva, Moskva **13**, 267–275 (1961).

[3] Landolt-Börnstein, *Zahlenwerte und Funktionen aus Naturwissenschaften und Technik*, Gruppe II, Band 5, Springer Verlag, Berlin 1967.

[4] W. Schaaffs, *Molekularakustik*, Springer Verlag, Berlin 1963.

[5] W. Schaaffs, R. Kuhnkies, H. Woelk, *Die physikalische Deutung des molaren Schallvolumens und der darin enthaltenen Raoschen Regel*, Acustica **12**, 222–229 (1962).

[6] D. Sette, *Die Temperaturabhängigkeit des Stossfaktors in einer Formel für die Schallgeschwindigkeit in Flüssigkeiten*, Zeitschr. für Naturforschung **5A**, 170–171 (1950).

[7] E. Soczkiewicz, *Generalized Lennard-Jones potential and the acoustic properties of liquids*, PhD. Thesis, IPPT PAN, Warsaw 1973 (in Polish).

[8] E. Soczkiewicz, *Temperature and pressure changes of the collision factor in Schaaffs theory of ultrasound propagation in liquids*, Proceedings of the XXI Seminar on Acoustics, Rzeszów 1974, p. 153–157 (in Polish).

Received on 22nd December 1977

ARCHIVES OF ACOUSTICS
3, 4, 273-281 (1978)

ULTRASONIC AND HYPERSONIC INVESTIGATIONS OF VIBRATIONAL RELAXATION IN LIQUID THIOPHENE

PIOTR MIECZNIK

Chair of Acoustics, Adam Mickiewicz University,
60-769 Poznań, Matejki 48/49

The measurements of hypersonic wave propagation velocity in the frequency range 2-7 GHz and ultrasonic wave absorption coefficient in the frequency range 10-60 MHz in thiophene have revealed the existence of two relaxation regions associated with vibrational specific heat: the first one associated with the first mode (ν_1) of molecule vibrations and the second, associated with the remaining modes excluding the first one. It follows from the experimental values of hypersonic wave absorption coefficient that the bulk viscosity coefficient in this frequency range $\eta_{\gamma,\infty}$ exceeds by a factor 2.5 the laminar viscosity coefficient η_s.

1. Introduction

The results of previous investigations of vibrational relaxation in simple organic liquids such as benzene, toluene, thiophene and the like were analysed in terms of a single relaxation process [9, 10]. This approach was primarily based on the data on the ultrasonic wave absorption coefficient. On the other hand, Hunter [5], who measured the values of ultrasonic wave absorption coefficient using pulse technique and the values of hypersonic wave propagation velocity by stimulated Mandelstam-Brillouin scattering, came to a conclusion that there exist two relaxation processes in benzene: the first one associated with the specific heat of all types of vibrational modes except the first mode and the second – associated with the specific heat of the first mode Takagi [13] came to similar conclusions on the basis of the data on hypersonic wave propagation velocity in benzene in the frequency range 2–5 GHz.

In his theory Hunter [5] argues (cf. also Kleszczewski [6]) that the results based on the measurement of ultrasonic wave absorption coefficient do not include relaxation of all vibrational modes since the assumption of a single relaxation time leads to the description which is inconsistent with the results of velocity measurements in hypersonic range (Mandelstam-Brillouin scattering).

A consistent interpretation of the results on ultrasonic absorption and hypersonic wave propagation velocity is feasible in terms of two relaxation regions.

Beste [1, 2] attributes the relaxation process in the vapours of benzene and of its derivatives, i.e. in C_6H_5F, C_6H_5Cl, C_6H_5Br, C_6H_5J to the specific heat of a vibrational mode of the lowest frequency. From the determined mean collision time he determines the transition probability for vibrational quanta. This probability decreases with an increase of molecule vibration frequency [2]. Each vibrational mode can be associated with a single relaxation time. However, ultrasonic measurements reveal the existence of a single relaxation process, also in the case of compound molecules. In the case of a continuous excitation, such as at generation of acoustic waves in the medium, a strong coupling of the modes occurs. Therefore the relaxation process will proceed through excitation of the modes of the lowest frequency. The relaxation time of these modes and their frequency both decrease with an increase of molecular weight [2].

The present paper describes the investigations of thiophene in the range of hypersonic frequencies. The measurements of hypersonic wave propagation velocity made it possible to draw conclusions on the nature of vibrational relaxation in thiophene.

The measurements of ultrasonic wave absorption coefficient in thiophene, shown in Fig. 1, indicate that in the frequency range 20-5000 MHz a single relaxation process occurs with a relaxation time $\tau = 5.7 \times 10^{-10}$ s.

These results made it possible to determine the acoustic relaxation contribution to the vibrational specific heat, amounting to $C_{ak} = 35.58$ J(mole·deg).

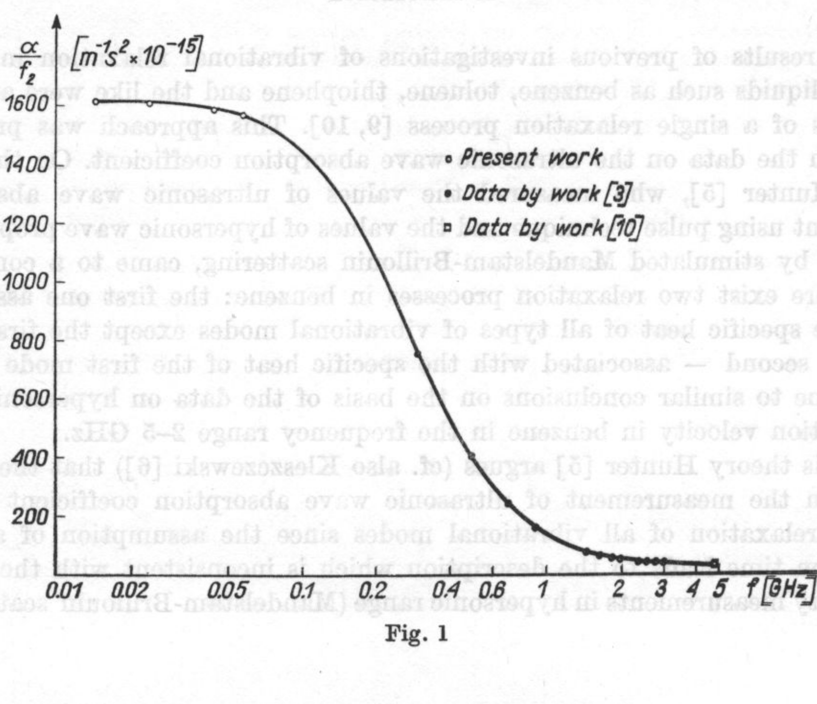


Fig. 1

On the other hand, the analysis of the results of the investigations of molecular spectra and the Planck-Einstein formula

$$C_{\mathrm{osc}} = \sum g_i \frac{(h\nu_i/kT)^2}{e^{h\nu_i/kT}(1-e^{-h\nu_i/kT})}, \quad (1)$$

where g_i is the degeneracy multiplicity, h – the Planck constant, k – the Boltzmann constant, T – the absolute temperature, ν_i – the i-th frequency of molecule modes, makes it possible to calculate the part of specific heat associated with intramolecular oscillations C_{osc}. The frequencies of vibrational modes for thiophene [12] are summarized in Table 1.

Table 1. Wave numbers of vibrational modes in thiophene [cm^{-1}]

ν_1	ν_2	ν_3	ν_4	ν_5	ν_6	ν_7	ν_8	ν_9	ν_{10}	ν_{11}	ν_{12}	ν_{13}	ν_{14}
450	603	710	748	832	1031	1078	1250	1357	1408	1588	1773	2998	3110

The value for thiophene calculated in this way equals to $C_{\mathrm{osc}} = 41.149$ J/mole deg. The difference $C_{\mathrm{osc}} - C_{\mathrm{ak}} = 5.5674$ J/(mole. deg). Thus, a past of the specific heat is associated with relaxation at a higher frequency. Following the Hunter's assumption this is the frequency of the first type of vibrational mode, which in thiophene amounts to $\nu_1 = 450\ \mathrm{m}^{-1}$ ($13.56\cdot 10^{12}$ Hz). The specific heat for this type of vibration amounts to $C_{\mathrm{osc}} = 5.599$ J/(mole·deg), and thus equals to the difference $C_{\mathrm{osc}} - C_{\mathrm{ak}}$.

The aim of the paper was thus to find the second relaxation range in the range of high frequencies, corresponding to the first type of vibrational mode ν_1.

2. Theory

The general form of the velocity dispersion equation for a single vibrational relaxation process [4] is

$$v^2 = v_0^2\left\{1+\frac{C_p-C_v}{C_p}\,\frac{C_{\mathrm{osc}}}{C_v-C_{\mathrm{osc}}}\,\frac{(f/f_r)^2}{1+(f/f_r)^2}\right\}, \quad (2)$$

where C_{osc} is the specific heat of all types of oscillation modes, calculated from (1), v – acoustic wave propagation velocity, v_0 – acoustic wave propagation velocity for $f\to 0$, f – frequency, f_r – relaxation frequency, C_p – specific heat at constant pressure, C_v – specific heat at constant volume.

The value of the velocity at the highest frequencies follows from equation (2) under the assumption $f \gg f_r$. Then

$$v_\infty^2 = v_0^2\,\frac{C_v(C_p - C_{\mathrm{osc}})}{C_p(C_v - C_{\mathrm{osc}})}. \quad (3)$$

On the other hand, if the process is described by two relaxation times, one can write [13]

$$v^2 = v_0^2\left\{1+\frac{C_p-C_v}{C_p}\cdot\frac{C_1}{C_v-C_1}\cdot\frac{(f/f_1)^2}{1+(f/f_1)^2}+\right.$$
$$\left.+\frac{C_p-C_v}{C_p}\cdot\frac{C_2}{C_v-C_1}\cdot\frac{C_v}{C_v-C_{osc}}\cdot\frac{(f/f_2)^2}{1+(f/f_2)^2}\right\}, \quad (4)$$

where C_1 is the past of the specific heat associated with the relaxation at the frequency f_1, C_2 – the part of the specific heat associated with relaxation at the frequency $f_2 > f_1$, whereas $C_{osc} = C_1 + C_2$.

The theory of double relaxation, proposed by Hunter, assumes that the first type of vibrational mode is associated with relaxation at a higher frequency f_2 and the remaining types of modes-with relaxation at a lower frequency f_1.

If two relaxation processes are sufficiently separated on the frequency scale, one can write an expression for the intermediate velocity values expected above the frequency corresponding to the process of first single relaxation. By virtue of equation (4) we have

$$v_i^2 = v_0^2\,\frac{C_v(C_p-C_1)}{C_p(C_v-C_1)}, \quad (5)$$

while, in region $f \cong f_1$, equation (4) assumes the form

$$\frac{v_i^2-v_0^2}{v_i^2-v^2} = 1+\left(\frac{f}{f_2}\right)^2. \quad (6)$$

It follows from this equation that the dependence of $(v_i^2-v_0^2)/(v_i^2-v^2)$ on f^2 is a straight line with a slope $(1/f_1^2)$. This relation makes it possible to estimate the first relaxation frequency f_1.

A similar equation for higher frequencies,

$$\frac{v_\infty^2-v_i^2}{v_\infty^2-v^2} = 1+\left(\frac{f}{f_2}\right)^2, \quad (7)$$

makes it possible to determine f_2.

The relaxation forces which are a measure of the influence of vibrational degrees of freedom are determined [13] by the relations

$$v_1 = \frac{v_i^2-v_0^2}{v_i^2} = \frac{C_p-C_v}{C_v}\cdot\frac{C_1}{C_p-C_1}, \quad (8)$$

$$v_2 = \frac{v_\infty^2-v_1^2}{v_\infty^2} = \frac{C_p-C_v}{C_v-C_1}\cdot\frac{C_2}{C_p-C_{osc}}, \quad (9)$$

respectively, for particular relaxation processes.

3. Experiment

The hypersonic wave propagation velocity was determined from the shift of the components of the fine structure of the light scattered in the Mandelstam-Brillouin scattering process. The detection of light scattered at any angle mode it possible to measure the hypersonic wave propagation velocity at various frequencies. The measurement system is shown schematically in Fig. 2.

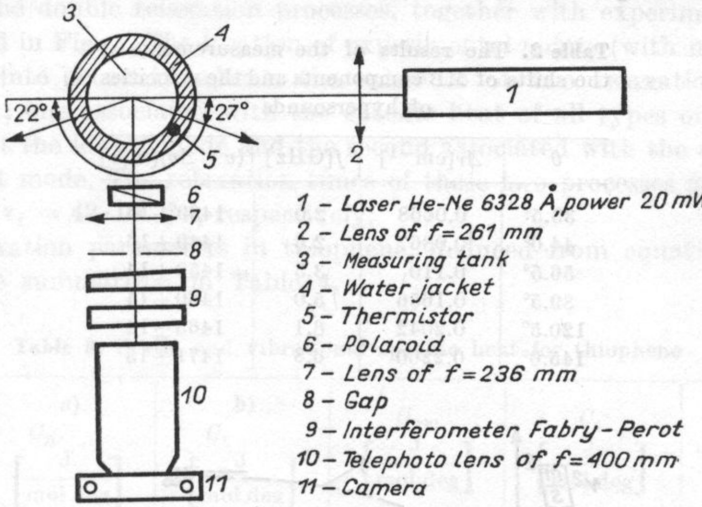


Fig. 2

The hypersound velocity was determined from the formula

$$v_h = \frac{\Delta\nu\, c\, \lambda}{2n_D \sin\theta/2}, \tag{10}$$

where $\Delta\nu$ is the change of light-wave frequency in cm^{-1}, C – the velocity of light, λ – wave length of the light, n_D – refraction index for light, θ – scattering angle.

The hypersound velocity was measured with an accuracy of $1\,\%$. The exposure time of photographic plate was about 1.5 h. The distance between the mirrors of the Fabry-Perot interferometer was fixed at $d = 10$ mm. The registered spectrum of scattered light was scanned photometrically using an automatic IFO 451 microphotometer. The investigated liquid specified as analytically pure was additionally distilled twice. The temperature of the liquid was kept constant with an accuracy of 0.1°C and monitored using a platinum temperature sensor.

The measurements of ultrasonic wave absorption coefficient in the range 10-60 MHz were performed using the US-4 High-Frequency Set manufacture by the Institute of Fundamental Technological Research of the Polish Academy of Sciences.

4. Experimental results and discussion

The results of measurements of hypersound velocity in thiophene at $t = 20°C$ are summarized in Table 2.

Under the assumption of a single relaxation time the relation $v^2 = f(f)$ was plotted (Fig. 3). The values of the velocity of ultrasonic wave propagation at the lowest frequency v_0 was obtained from (2) and the values of the velocity at the highest frequency v_∞ from (3). The value of relaxation frequency was

Table 2. The results of the measurements of the shifts of MB components and the velocities of hypersounds

θ	$\Delta\nu[\mathrm{cm}^{-1}]$	f[GHz]	$(v \pm \Delta v)$[m/s]
33.5°	0.0668	2.0	1439 ± 15
44.0°	0.0867	2.6	1449 ± 16
56.5°	0.110	3.3	1450 ± 14
89.5°	0.1656	5.0	1460 ± 14
120.5°	0.2042	6.1	1465 ± 14
145.0°	0.2260	6.8	1471 ± 15

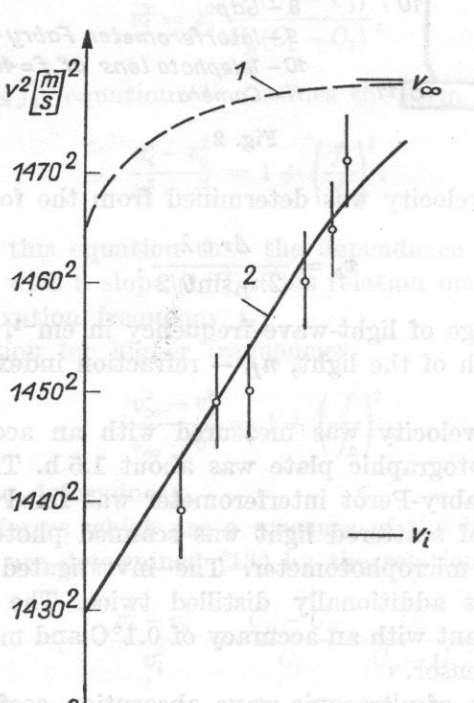


Fig. 3

assumed to be $f_r = 280$ MHz on the basis of the relation $\alpha/f_2 = f(f^2)$ presented in Fig. 1. The data from Table 3 were used.

The theory of double relaxation made it possible to determine relaxation frequency f_2 in the high-frequency range from the dependence of $(v_\infty^2 - v_i^2)(v_\infty^2 - v^2) - 1$ on f^2.

By making use of equation (4) the theoretical relation $v^2 = f(f)$ was plotted assuming double relaxation. Both theoretical relations $v^2 = f(f)$ for the single and the double relaxation processes, together with experimental points are presented in Fig. 3. The location of experimental points (with measurement error taken into accuont) indicate the existence of two relaxation processes in thiophene; one associated with the specific heat of all types of vibrational modes except the lowest mode and the second associated with the specific heat of the lowest mode. The relaxation times of these two processes are $\tau_1 = 5.7 \cdot 10^{-10}$ s and $\tau_2 = 42 \cdot 10^{-10}$ s, respectively.

The relaxation parameters in thiophene, deduced from equations (7), (9) and (10) are summarized in Table 4.

Table 3. Static and vibrational specific heat for thiophene

T [°C]	C_p a) $\left[\frac{J}{mol\ deg}\right]$	C_v b) $\left[\frac{J}{mol\ deg}\right]$	C_{osc} $\left[\frac{J}{mol\ deg}\right]$	C_1 $\left[\frac{J}{mol\ deg}\right]$	C_2 $\left[\frac{J}{mol\ deg}\right]$
20	125.106	84.319	41.149	35.550	5.599

a) – from [8], p. 528; b) – from the ratio $C_p/C_v = 1.46$ cited in [11].

Table 4. Relaxation parameters in thiophene

(a)

T [°C]	v_0 [m/s]	v_i [m/s]	v_∞ [m/s]	$\left(\frac{\alpha}{f^2}\right)_0$ [10^{-15} m^{-1} s^2]	$\left(\frac{\alpha}{f^2}\right)_\infty$ [10^{-15} m^{-1} s^2]
20	1296	1437	1478	1600	22

(b)

T [°C]	f_1 [MHz]	r_1	f_2 [MHz]	r_2
20	280	0.187	3800	0.0543

5. Conclusions

The results of the measurements of hypersonic wave propagation velocity and ultrasonic wave absorption coefficient in thiophene reveal the existence of two relaxation processes. A comparison of the values of the specific heats obtained from an analysis of the results of the investigations of molecular spectra (C_{osc}) and the values obtained from acoustical measurements (C_{ak}) indicates that the first relaxation process in the range of ultrasonic frequencies is associated with the specific heat of all types of vibrational modes except the lowest frequency mode ν_1, whereas the second one is associated with the specific heat of the lowest frequency mode.

The existence of the second relaxation process in thiophene is confirmed by the value of the ratio of bulk viscosity coefficient at hypersonic frequencies $\eta_{v,\infty}$ to the laminar viscosity coefficient η_s. The bulk viscosity coefficient has been determined from the formula

$$\left(\frac{\alpha}{f_2}\right)_\infty = \frac{2\pi^2(4\eta_s/3+\eta_{v,\infty})}{\varrho v_0^3}. \tag{11}$$

It has been assumed [11] that $(\alpha/f^2)_\infty = 22\cdot 10^{-15}$ [m^{-1} s^2], laminar viscosity coefficient $\eta_s = 664\cdot 10^{-6}$ [m^{-1} kg s^{-1}], density $\varrho = 1.0644\cdot 10^3$ [kg/m^3]; $v_0 = 1296$ [m/s]. All above values refer to $t = 20°$C. The value of $\eta_{v,\infty}$, calculated from equation (11), makes it possible to estimate the ratio $\eta_{v,\infty}/\eta_s$ which turns out to be 2.5. This is greater than unity and thus indicates the existence of a relaxation process in the hypersonic range. Its character has been described above.

References

[1] K. W. Beste, *Eigenschwingungen von Benzol und Benzol-halogeniden und die durch Ultraschallabsorbtionsmessungen bestimmten Übergangswahrscheinlichkeiten für Schwingungsquanten*, Acustica, **33**, 2, 122–126 (1975).

[2] — *Ultraschallapparatur zur Messung von Relaxationserscheinungen organischer Verbindungen*, Acustica, **31**, 14–21 (1974).

[3] L. A. Dawidowicz, S. Machkamow, Ł. Pułatowa, P. K. Habibułłajew and M. G. Haliulin, *Issledovanie akusticzeskich swojstw niekotorych organiczeskich żidkostiej na czastotach* 0,3–3 GHz, Akusticzeskij Żurnał, **18**, 318–320 (1972).

[4] K. F. Herzfeld and T. A. Litowitz, *Absorption and dispersion of ultrasonic waves*, Academic Press Inc., New York 1959.

[5] J. L. Hunter, E. F. Carone, H. D. Dardy and J. A. Bucaro, *Ultrasonic and hypersonic studies of vibrational relaxation*, J. Acoust. Soc. Amer., **40**, 2, 313–316 (1966).

[6] Z. Kleszczewski, *Thermal relaxation processes in some organic liquids*, Archiwum Akustyki, **11**, 1, 59–66 (1976).

[7] Landolt-Börnstein, *Physikalisch-chemische Tabellen I*, Springer Verlag, Berlin 1923, 134.

[8] Landolt-Börnstein, *Zahlenwerte und Funktion aus Physik, Chemie*, Springer Verlag, Berlin 1961.

[9] P. Mason, *Physical Acoustics II A*, New York and London 1965.

[10] C. L. O'Connor, J. P. Schlupf, *Brillouin Scattering and Thermal Relaxation in Benzene*, J. Acoust. Soc. Amer., **40**, 663–666 (1966).

[11] D. A. Rasułmuhamedowa, H. Rihsitiłłajew, T. Idirow, M. G. Haliulin, A. Rasułmuhamedow, P. Habibułłajew, *Issliedowanie tonkoj struktury liniirelejowskowo rassiejanija swieta w niekotorych organiczeskich żidkostiach*, Optika i spektroskopia, **32**, 6, 1116–1117 (1972).

[12] B. Schräder, W. Meier, *Roman/IR Atlas organischer Verbindungen*, 2, Groups F-O s. J6–O1.

[13] K. Takagi, P. K. Choi, Negishi, *Ultrasonic and Hypersonic Studies of relaxation in liquid Benzene*, Acoustica, **34**, 5, 336–340 (1976).

Received on 30th November 1977

ARCHIVES OF ACOUSTICS
3, 4, 283-292 (1978)

CORRELATION FUNCTION DETERMINATION FOR INHOMOGENEITIES SCATTERING AN ACOUSTIC WAVE

JÓZEF LEWANDOWSKI

Department of Physical Acoustics, Institute of Fundamental Technological Research, Polish Academy of Sciences (Warsaw)

A random inhomogeneous isotropic medium filling a domain immersed in an infinitely extended homogeneous isotropic medium is considered. The formulae describing the scalar potential of the scattered field are deduced for small and large distances from the domain of the heterogeneous material. The fluctuations of density and wave propagation velocity (and also pressure in the case of a nonviscous emulsion) are treated as random variables of the space coordinates. The correlation function is calculated from the appropriate farfield solution and expressed in terms of a scalar potential for the angular distribution of the scattered wave. This general method is adapted for a non-viscous random emulsion and the correlation function is expressed in terms of the intensity angular distribution of the scattered wave.

List of symbols

V' — domain filled by the inhomogeneous medium
S' — boundary of the domain V'
$\boldsymbol{r}$ — position vector
$\psi(\boldsymbol{r}, t)$ — scalar velocity potential of an acoustic wave
ϱ_s and $\varrho_0(\boldsymbol{r})$ — density of the homogeneous and heterogeneous medium, respectively
c_s and $c_0(\boldsymbol{r})$ — wave velocity in the homogeneous and heterogeneous medium, respectively
$\langle A \rangle$ — mean value of a quantity A
$\delta A(\boldsymbol{r}) = A(\boldsymbol{r}) - \langle A(\boldsymbol{r}) \rangle$ — fluctuation in a quantity A at a point $\boldsymbol{r}$
$\gamma(x)$ — autocorrelation function (called shortly correlation function)
L_c — correlation length
β — volume concentration of the grains
$\bar{F}$ — equilibrium value of a quantity F
ΔF — acoustic disturbance of a quantity F
ω — angular frequency
a — amplitude of oscillations
$\boldsymbol{n}$ — unit vector in the direction of propagation of the incident wave
θ — angle of scattering
p — pressure
η — kinematic viscosity
I_0 — intensity of the incident wave
I_{sc} — intensity of the scattered wave

1. Introduction

Several authors [1, 2, 5, 6] have considered the problem of finding the scattered intensity and the scattering cross section in a random inhomogeneous isotropic media from the incident acoustic wave and the correlation function [4]. In these previous works the scattering intensities have been calculated from the appropriate farfield solutions, and the results have been expressed in terms of correlation functions. The purpose of the present paper is to solve the inverse problem of determining the correlation function from the angular distribution of the acoustic field of a wave scattered in a random inhomogeneous isotropic medium. The inverse problem under consideration is the analogue of the light scattering problem discussed by Debye and Bueche [4].

In the present paper we start with the differential equation of motion for the scalar wave $\psi(\boldsymbol{r}, t)$ in a heterogeneous medium, where the wave velocity $c_0(\boldsymbol{r})$ is a function of the position vector $\boldsymbol{r}$ and is independent of the time t. As a result we obtain an integral expression for the scalar scattered wave $\psi_{sc}(\boldsymbol{r}, t)$. With the help of the correlation function [4] and use of the Fourier integral transformation for odd functions we obtain, in the farfield approximation, a rather simple integral formula expressing the correlation function in terms of $\langle|\psi_{sc}(\boldsymbol{r}, t)|^2\rangle$ or $\langle|\nabla\psi_{sc}(\boldsymbol{r}, t)|^2\rangle$ and of the angle of scattering θ where $\langle\cdot\rangle$ denotes an average. Next we consider the special case of acoustic wave scattering in nonviscous emulsions. For this case we obtain an integral formula expressing the correlation function in terms of the scattered intensity and the angle of scattering.

2. Basic assumptions and auxiliary notions

In discussing the problem under consideration in this paper, the random heterogeneous isotropic material filling domain V' is assumed to be immersed in an infinitely extended homogeneous isotropic material of density ϱ_s, where the wave velocity c_s is known and satisfies the following inequality:

$$\left|1-\left(c_s\,|\,c_0(\boldsymbol{r})\right)^2\right| \equiv \left|1-\left[1+\left(c_0(\boldsymbol{r})-c_s\right)/c_s\right]^{-2}\right| \ll 1 \Leftrightarrow |c_0(\boldsymbol{r})-c_s|/c_s \ll 1. \tag{2.1}$$

Inequality (2.1) enables us to write with first order accuracy in

$$\left(c_0(\boldsymbol{r})-c_s\right)/c_s \equiv \left(\delta c_0(\boldsymbol{r})+\langle c_0(\boldsymbol{r})\rangle-c_s\right)/c_s,$$

the relation

$$U(\boldsymbol{r}) \equiv 1-\left(c_s/c_0(\boldsymbol{r})\right)^2 \cong \langle\tilde{U}(\boldsymbol{r})\rangle+\delta\tilde{U}(\boldsymbol{r}), \tag{2.2}$$

where

$$\tilde{U}(\boldsymbol{r}) = 2\left(c_0(\boldsymbol{r})-c_s\right)/c_s, \tag{2.3}$$

and

$$\delta\tilde{U}(\boldsymbol{r}) = 2\left(c_0(\boldsymbol{r})-\langle c_0(\boldsymbol{r})\rangle\right)/c_s. \tag{2.4}$$

Furthermore, it is assumed that

$$|\langle \tilde{U}(\boldsymbol{r})\rangle| \ll |\delta \tilde{U}(\boldsymbol{r})| \ll 1 \tag{2.5}$$

and

$$|\varrho_s c_s - \varrho_0(\boldsymbol{r}) c_0(\boldsymbol{r})| / \varrho_s c_s \ll 1, \tag{2.6}$$

where $\varrho_0(\boldsymbol{r})$ is the density of the medium filling domain V'.

The subsequent discussion uses a reference system with the origin at some convenient point of the domain V'. It is assumed that the volume V' of the domain filled by the inhomogeneities satisfies the inequality

$$(V')^{1/3} \gg L_c, \tag{2.7}$$

where L_c is the correlation lenght of the random inhomogeneities. The structure of the heterogeneous material filling the domain V' is described with the help of the correlation function $\gamma_{AB}(\boldsymbol{r}_1 - \boldsymbol{r}_2)$ which determines the manner in which the fluctuation δA in a quantity A at a given point $\boldsymbol{r}_1$ is correlated with that in another quantity B at a point $\boldsymbol{r}_2$. The fluctuation $\delta F(\boldsymbol{r}_0)$ in a quantity F at a point $\boldsymbol{r}_0$ is given by the formula

$$\delta F(\boldsymbol{r}_0) = F(\boldsymbol{r}_0) - \langle F(\boldsymbol{r}_0)\rangle. \tag{2.8}$$

When the material is isotropic, $\gamma_{AB}(\boldsymbol{r}_1 - \boldsymbol{r}_2)$ is a function of $|\boldsymbol{r}_1 - \boldsymbol{r}_2|$ and is independent of direction. The correlation function $\gamma_{AB}(\boldsymbol{r}_1 - \boldsymbol{r}_2)$ for an isotropic material is defined [4] by

$$\langle \delta A(\boldsymbol{r}_1)\, \delta B(\boldsymbol{r}_2)\rangle = \gamma_{AB}(x) \langle \delta A(\boldsymbol{r}_1)\, \delta B(\boldsymbol{r}_1)\rangle, \tag{2.9}$$

where

$$x = \boldsymbol{r}_1 - \boldsymbol{r}_2. \tag{2.10}$$

By comparing this equation with the condition

$$\lim_{x\to 0} \langle \delta A(\boldsymbol{r}_1)\, \delta B(\boldsymbol{r}_1 - x)\rangle = \langle \delta A(\boldsymbol{r}_1)\, \delta B(\boldsymbol{r}_1)\rangle \tag{2.11}$$

and defining the correlation length L_c by

$$\lim_{x\to L_c} \langle \delta A(\boldsymbol{r}_1)\, \delta B(\boldsymbol{r}_1 - x)\rangle = \langle \delta A(\boldsymbol{r})\, \delta B(\boldsymbol{r})\rangle / e, \tag{2.12}$$

we obtain:

$$\lim_{x\to 0} \gamma_{AB}(x) = 1, \quad \lim_{x\to L_c} \gamma_{AB}(x) = 1/e \quad (e = 2.718\ldots). \tag{2.13}$$

Formulae (2.9) and (2.13) are also applicable in the case of B being the same quantity as A. Then

$$\langle \delta A(\boldsymbol{r})\, \delta A(\boldsymbol{r} - x)\rangle \equiv \langle \delta A(0)\, \delta A(x)\rangle = \gamma(x) \langle (\delta A(\boldsymbol{r}))^2\rangle \tag{2.14}$$

and $\gamma(x)$ is called the *autocorrelation function*. $\gamma(x)$ measures the degree of correlation between the fluctuations at two points as a function of the distance of their separation.

The heterogeneous material considered is assumed to be a two-phase material. One of the phases consists of isolated grains randomly distributed in the matrix of the other phase in the domain V'. The fluctuations $\delta A(\boldsymbol{r})$, $\delta B(\boldsymbol{r})$, ... in quantities $A, B, \ldots$, respectively, are the result of fluctuations $\delta\beta(\boldsymbol{r})$ in the volume concentration $\beta(\boldsymbol{r})$ of the grains. For $\delta A(\boldsymbol{r})$ we have:

$$\delta A(\boldsymbol{r}) = \left(\frac{\partial A(\beta)}{\partial \beta}\right)_{\beta=0} \delta\beta \quad \text{if } \beta \ll 1. \tag{2.15}$$

Substituting equation (2.15) into (2.14) we obtain

$$\langle \delta A(0)\, \delta A(\boldsymbol{x})\rangle = \gamma(x) \left(\frac{\partial A}{\partial \beta}\right)^2_{\beta=0} \langle(\delta\beta)^2\rangle. \tag{2.16}$$

It can be verified that

$$\langle \delta A(0)\, \delta B(\boldsymbol{x})\rangle = \gamma(x) \left(\frac{\partial A}{\partial \beta}\right)_{\beta=0} \left(\frac{\partial B}{\partial \beta}\right)_{\beta=0} \langle(\delta\beta)^2\rangle. \tag{2.17}$$

Thus the autocorrelation function $\gamma(x)$ is adequate to describe all correlations if the fluctuations $\delta A(\boldsymbol{r})$, $\delta B(\boldsymbol{r})$, ... can be expressed in the form of equation (2.15). In this case an average of the type $\langle \delta A(0)\, \delta B(\boldsymbol{x})\rangle$ can be also reduced to the mean-square fluctuation $\langle(\delta\beta)^2\rangle$. As a result of the asumption (2.7) we have on the boundary S' of the domain V':

$$\gamma(x)|_{x\in S'} = 0, \qquad \left.\frac{\partial \gamma(x)}{\partial x}\right|_{x\in S'} = 0. \tag{2.18}$$

The acoustic wave under consideration in the present paper are assumed to be monochromatic, i.e. all the acoustic disturbances $\Delta F(\boldsymbol{r}, t)$ associated with the waves at a given point $\boldsymbol{r}$ are simple sinusoidal functions of time of the form

$$\Delta F(\boldsymbol{r}, t) = e^{i\omega t} \operatorname{const}(\boldsymbol{r}), \quad \Delta F = F - \overline{F}, \tag{2.19}$$

where ω is the angular frequency of the wave and $\overline{F}$ is the equilibrium value of a quantity F. It is assumed that

$$|\Delta F(\boldsymbol{r}, t)/\overline{F}(\boldsymbol{r})| \ll 1. \tag{2.20}$$

3. Angular distribution of the scalar scattered wave

The existence of a velocity scalar potential $\psi(\boldsymbol{r}, t)$ for an acoustic wave in the heterogeneous medium under consideration is postulated. The equation of motion for the scalar potential $\psi(\boldsymbol{r}, t)$ is postulated [7] to be

$$\nabla^2 \psi(\boldsymbol{r}, t) = \left(c_i(\boldsymbol{r}')\right)^{-2} \frac{\partial^2 \psi(\boldsymbol{r}, t)}{\partial t^2}, \tag{3.1}$$

where $i = 0$ if $\boldsymbol{r}$ is within the domain V', $i = s$ and $c_s(\boldsymbol{r}) = c_s = \text{const}$ it $\boldsymbol{r}$ is outside the domain V' or on the boundary S' of V'.

By substituting

$$\psi(\boldsymbol{r}, t) = \varphi(\boldsymbol{r})e^{i\omega t} \equiv \varphi(\boldsymbol{r})e^{ik_s c_s t}, \quad k_s = \omega/c_s, \tag{3.2}$$

we obtain the equation

$$(\nabla^2 + k_s^2)\varphi(\boldsymbol{r}) = k_s^2 U(\boldsymbol{r})\varphi(\boldsymbol{r}), \tag{3.3}$$

which the function $\varphi(\boldsymbol{r}')$ must satisfy. $U(\boldsymbol{r})$ is given by formula (2.2); $U(\boldsymbol{r})$ being different from zero only if $\boldsymbol{r}$ is within the domain V'. Furthermore, the scalar potential $\psi(\boldsymbol{r}, t)$ is taken as the sum of a monochromatic travelling plane wave (primary or incident wave) with another one superposed (called the *scattered wave*).

The problem formulated above, of calculating $\varphi(\boldsymbol{r})$ when $\boldsymbol{k}_s$ and $U(\boldsymbol{r})$ are known, is the same, apart from the factor k_s^2 on the right-hand side of equation (3.3), as the quantum mechanical problem of finding de Broiglie waves connected with the stationary elastic scattering of spinless particles. On using a suitable method (e.g. a Green's function method) [3], the solution of equation (3.3) is found to be

$$\varphi(\boldsymbol{r}) = e^{i\boldsymbol{k}_s\cdot\boldsymbol{r}} + \varphi_{sc}(\boldsymbol{r}), \quad \boldsymbol{k}_s = k_s\boldsymbol{n}, \tag{3.4}$$

where $\boldsymbol{n}$ is the unit vector in the direction of propagation of the incident wave $e^{i\boldsymbol{k}_s\cdot\boldsymbol{r}}$. The incident wave is the solution of equation (3.3) for $U(\boldsymbol{r}) = 0$. Assuming that

$$|e^{i\boldsymbol{k}_s\cdot\boldsymbol{r}}| \gg |\varphi_{sc}(\boldsymbol{r})|, \tag{3.5}$$

we obtain for the scalar potential of the scattered wave the formula

$$\varphi_{sc}(\boldsymbol{r}) = (1/r)A(\boldsymbol{r})e^{ik_s r}, \tag{3.6}$$

where

$$A(\boldsymbol{r}) = -\frac{k_s^2 r}{4\pi}\int_{V'} \frac{[U(\boldsymbol{r}')e^{i\boldsymbol{k}_s\cdot\boldsymbol{r}'}]e^{ik_s(|\boldsymbol{r}-\boldsymbol{r}'|-r)}}{|\boldsymbol{r}-\boldsymbol{r}'|}d^3\boldsymbol{r}', \tag{3.7}$$

the integration being over the volume of element

$$dV' \equiv d^3\boldsymbol{r}' \equiv dx'\,dy'\,dz'$$

in the scattering domain V'. By $\boldsymbol{r}$ we denote the position vector of a point within the domain V', while by $\boldsymbol{r}$ – the position vector of an "observation point" ($\boldsymbol{r}$ may be within the domain V' as well as outside this domain). The additional assumption of the "farfield" approximation

$$k_s(r')^2/2r \ll 1, \quad |\boldsymbol{r}'| \ll |\boldsymbol{r}|, \tag{3.8}$$

enables us to write the expression for $A(\boldsymbol{r})$ as

$$A(\boldsymbol{r}) = -\frac{k_s^2}{4\pi}\int_{V'} U(\boldsymbol{r}')\, e^{i\boldsymbol{K}\cdot\boldsymbol{r}'}\, d^3\boldsymbol{r}', \tag{3.9}$$

where

$$\boldsymbol{K} = [\boldsymbol{n} - (\boldsymbol{r}/r)]k_s, \quad K = 2k_s \sin(\theta/2), \tag{3.10}$$

θ being the angle between the vectors $\boldsymbol{r}$ and $\boldsymbol{k}_s$ (the angle of scattering). Condition (3.8) means that we are considering only the solution valid outside the domain V' at large distances r from the inhomogeneities.

Assumptions (2.1), (2.5) and formulae (2.2), (2.3), (2.4), (2.15) enable us to write, with first order accuracy,

$$\langle U(\boldsymbol{r}_1')\, U(\boldsymbol{r}_2')\rangle = \langle \delta\tilde{U}(\boldsymbol{r}_1')\, \delta\tilde{U}(\boldsymbol{r}_2')\rangle = \gamma(x)\langle (\delta\tilde{U}(\boldsymbol{r}_1'))^2\rangle, \quad \boldsymbol{x} = \boldsymbol{r}_1' - \boldsymbol{r}_2', \tag{3.11}$$

where

$$\langle (\delta\tilde{U}(\boldsymbol{r}'))^2\rangle = \left(\frac{\partial \tilde{U}}{\partial \beta}\right)^2_{\beta=0} \langle (\delta\beta)^2\rangle \quad \text{if} \quad \beta \ll 1 \tag{3.12}$$

From (3.6), (3.9), (3.10), (3.11) it follows that

$$\langle |\varphi_{sc}(\boldsymbol{r})|^2\rangle = \frac{B}{r^2}\int_{V'} \gamma(x)\, e^{i\boldsymbol{K}\boldsymbol{x}}\, d^3\boldsymbol{r}_1'\, d^3\boldsymbol{r}_2', \tag{3.13}$$

where

$$B = k_s^4 \langle (\delta\tilde{U}(\boldsymbol{r}'))^2\rangle / 16\pi^2. \tag{3.14}$$

By introducing the new variables

$$\boldsymbol{r}_0 \equiv (x_0, y_0, z_0) = \boldsymbol{r}_1' - \boldsymbol{x}/2 = \boldsymbol{r}_2' + \boldsymbol{x}/2,$$

the integrations over x_0, y_0, z_0 and over all directions can be performed. Performing these integrations ($\boldsymbol{K}$ being the polar axis), and using the Fourier integral transformation for odd functions, we obtain:

$$\gamma(x) = (r^2/2\pi^2 B V')\int_0^\infty \langle |\varphi_{sc}|^2\rangle K^2 \frac{\sin Kx}{Kx}\, dK. \tag{3.15}$$

Using (2.13) and the well-known formula

$$\lim_{x\to 0} \frac{\sin Kx}{Kx} = 1, \tag{3.16}$$

we obtain finally

$$\gamma(x) = \left[\int_0^\infty \langle |\varphi_{sc}|^2\rangle K^2 \frac{\sin Kx}{Kx}\, dK\right] \cdot \left[\int_0^\infty \langle |\varphi_{sc}|^2\rangle K^2 dK\right]^{-1}, \tag{3.17}$$

$$\omega = \text{const}, \quad r = \text{const}.$$

Using formulae (3.6), (3.9), (3.10) the expression for $\nabla\varphi_{sc}(\mathbf{r})$ can be found. Finding this expression and performing the same mathematical operations which had given equation (3.17) from equations (3.6), (3.9), (3.10), we obtain finally

$$\gamma(x) = \Big[\int_0^\infty \langle|\nabla\varphi_{sc}|^2\rangle K^2 \frac{\sin Kx}{Kx}\, dK\Big]\cdot\Big[\int_0^\infty \langle|\nabla\varphi_{sc}|^2\rangle K^2 dK\Big]^{-1}, \tag{3.18}$$

$$\omega = \text{const}, \ r = \text{const}.$$

Formulae (3.17) and (3.18) permit us to determine the correlation function $\gamma(x)$ from the angular distribution of the scalar potential and the gradient of the scalar potential of the wave scattered by a random inhomogeneous isotropic medium, respectively.

4. Angular distribution of the intensity of the acoustic wave scattered by a nonviscous emulsion

The case of a nonviscous emulsion will be considered as an example of a two-phase random heterogeneous material filling the domain V'. In the present model an emulsion is considered as a mixture of two chemically non reacting and nonviscous fluids, one of which is not soluble in the another. One fluid is coherent and volumetrically dominant and the other is dispersed in the forms of grains randomly distributed in the matrix fluid. The fluctuations $\delta c_0(\mathbf{r}')$ and

$$\delta\bar{\varrho}_0(\mathbf{r}') = \bar{\varrho}_0(\mathbf{r}') - \langle\bar{\varrho}_0(\mathbf{r}')\rangle \tag{4.1}$$

(where $\bar{\varrho}_0(\mathbf{r}')$ is the equilibrium value of the density $\varrho_0(\mathbf{r}')$ within the domain V') are the results of fluctuations $\delta\beta$ in the volume concentration β of the grains. In accordance with the basic assumptions of the present paper, the emulsion filling the domain V' is assumed to be immersed in an infinitely extended fluid of density ϱ_s, where the wave velocity $c_s(\mathbf{r}) = c_s = \text{const}$ is known. It is thus also assumed that inequalities (2.1), (2.5) and (2.6) are valid. Furthermore, it is assumed that

$$|\delta\varrho_0(\mathbf{r}')|/\langle\bar{\varrho}_0(\mathbf{r}')\rangle \ll 1. \tag{4.2}$$

The linearized acoustic equations of the system under consideration may be obtained from the general equations of flow, by omitting all the higher order terms in small acoustic disturbances. The acoustic disturbances under consideration in the present paper are assumed to be the periodic fluctuations, of the form given by equations (2.19)–(2.20), in the density $\Delta\varrho(\mathbf{r},t)$, pressure $\Delta p(\mathbf{r},t)$ and the velocity $\mathbf{v}(\mathbf{r},t)$ of the liquid about the equilibrium values

$$\bar{\varrho}(\mathbf{r},t) = \bar{\varrho}(\mathbf{r}), \quad \bar{p}(\mathbf{r},t) = p_0 = \text{const}, \quad \bar{\mathbf{v}}(\mathbf{r},t) = 0, \tag{4.3}$$

respectively. The fluctuations are assumed to be adiabatic, i.e.

$$\frac{dp(\boldsymbol{r},t)}{dt} = c_i^2(\boldsymbol{r})\frac{d\varrho(\boldsymbol{r},t)}{dt}, \tag{4.4}$$

subject to the rules given under equation (3.1) for $i = 0$ or $i = s$.

It is assumed [1] that the equations of flow in the case under consideration have the following form:

$$\frac{d\varrho(\boldsymbol{r},t)}{dt} + \varrho(\boldsymbol{r},t)\operatorname{div}\boldsymbol{v}(\boldsymbol{r},t) = 0, \tag{4.5}$$

$$\varrho(\boldsymbol{r},t)\frac{d\boldsymbol{v}(\boldsymbol{r},t)}{dt} + \nabla p(\boldsymbol{r},t) = 0. \tag{4.6}$$

In order to determine the range of applicability of the nonviscous emulsion approximation given by equations (4.5) and (4.6), we have to introduce appropriate dimensionless variables into Navier-Stokes equation. In this way it can be verified that the nonviscous emulsion approximation is justified if

$$L_c^2\omega/\eta \gg 1, \quad L_c\langle c_0(\boldsymbol{r}')\rangle/\eta \gg 1, \quad 2\pi\langle c_0(\boldsymbol{r}')\rangle/\omega \gg a, \tag{4.7}$$

where η is the kinematic viscosity and a is the amplitude of the oscillations. Combining equations (4.4), (4.5) and (4.6) we obtain [1] the first order acoustic equation

$$\nabla^2\big(\Delta p(\boldsymbol{r},t)\big) = \big(c_i(\boldsymbol{r})\big)^{-2}\frac{\partial^2\Delta p(\boldsymbol{r},t)}{\partial t^2} + \nabla\ln\bar{\varrho}(\boldsymbol{r})\cdot\nabla\big(\Delta p(\boldsymbol{r},t)\big), \tag{4.8}$$

where $c_i(\boldsymbol{r}) = c_0(\boldsymbol{r}')$ and $\nabla\ln\bar{\varrho}(\boldsymbol{r}) \neq 0$ if $\boldsymbol{r}$ is within the domain V', and $c_i(\boldsymbol{r}) = c_s = \text{const}$ and $\nabla\ln\bar{\varrho}(\boldsymbol{r}) = 0$ if $\boldsymbol{r}$ is outside the domain V' or on the boundary of V'.

By substituting

$$\Delta p(\boldsymbol{r},t) = \Delta p(\boldsymbol{r})e^{i\omega t} = \Delta p(\boldsymbol{r})e^{ik_s c_s t}, \tag{4.9}$$

we obtain

$$(\nabla^2 + k_s^2)\Delta p(\boldsymbol{r}) = k_s^2 U_p(\boldsymbol{r})\Delta p(\boldsymbol{r}), \tag{4.10}$$

where $U_p(\boldsymbol{r})$ is given, to the first order, by the expression

$$U_p(\boldsymbol{r}) = U(\boldsymbol{r}) + \big(1/k_s^2\langle\bar{\varrho}(\boldsymbol{r})\rangle\big)\,\nabla\big(\delta\bar{\varrho}(\boldsymbol{r})\big)\cdot\nabla, \tag{4.11}$$

k_s and $U(\boldsymbol{r})$ are given by formulae (3.2) and (2.2), respectively.

The pressure disturbance $\Delta p(\boldsymbol{r},t)$ is taken as a sum of a monochromatic travelling plane wave (the incident wave) $P_0e^{i(\omega t + k_s r)}$ superposed on another, called the *scattered wave,* $\Delta p_{sc}(\boldsymbol{r})e^{i\omega t}$. Thus the solution of equation (4.10) is taken as

$$\Delta p(\boldsymbol{r}) = P_0e^{i\boldsymbol{k}_s\cdot\boldsymbol{r}} + \Delta p_{sc}(\boldsymbol{r}). \tag{4.12}$$

The incident wave $P_0 e^{i\boldsymbol{k}_s\cdot\boldsymbol{r}}$ is the solution of equation (4.10) for the case of $U_p(\boldsymbol{r}) = 0$. The assumption

$$|\Delta p_{sc}(\boldsymbol{r})| \ll |P_0 e^{i\boldsymbol{k}_s\cdot\boldsymbol{r}}| \tag{4.13}$$

leads to

$$\Delta p_{sc}(\boldsymbol{r}) = (1/r) A_p(\boldsymbol{r}) e^{ik_s r}, \tag{4.14}$$

where $A_p(\boldsymbol{r})$ is given by formula (3.7) if $U(\boldsymbol{r})$ is replaced by $U_p(\boldsymbol{r})$, and $e^{i\boldsymbol{k}_s\cdot\boldsymbol{r}'}$ is replaced by $P_0 e^{ik_s r'}$. Using the assumptions (2.1), (2.5), (2.6), and (3.8), we arrive at the approximate (to first order) integral expression

$$A_p(\boldsymbol{r}) = \frac{-k_s^2 P_0}{4\pi} \int_{V'} \left[2\frac{\delta c_0(\boldsymbol{r}')}{c_s} + \frac{i\boldsymbol{k}_s}{\langle\bar{\varrho}(\boldsymbol{r}')\rangle k_s} \cdot \nabla\big(\delta\bar{\varrho}(\boldsymbol{r}')\big)\right] e^{i\boldsymbol{k}\cdot\boldsymbol{r}'} d^3 r, \tag{4.15}$$

where $\boldsymbol{K}$ is given by formula (3.10). These same assumptions together with the assumptions of (2.18) and formulae (2.16), (3.11) and (3.12) enable us to write, with first order accuracy,

$$\langle|\Delta p_{sc}(\boldsymbol{r})|^2\rangle = \frac{B_p}{r^2} \int_{V'} \gamma(x) e^{i\boldsymbol{K}\cdot\boldsymbol{x}} d^3 \boldsymbol{x}, \tag{4.16}$$

where

$$B_p = \frac{k_s^4 P_0^2 V'}{16\pi^2} \langle(\delta\beta)^2\rangle \left[\frac{2}{c_s}\left(\frac{\delta c_0(\boldsymbol{r}')}{\partial\beta}\right)_{\beta=0} + \frac{\boldsymbol{k}_s\cdot\boldsymbol{K}}{k_s^2\langle\bar{\varrho}(\boldsymbol{r}')\rangle}\left(\frac{\partial\bar{\varrho}(\boldsymbol{r}')}{\partial\beta}\right)_{\beta=0}\right]^2, \tag{4.17}$$

and $\boldsymbol{x}$ is given by formula (3.11). In the case of liquids (emulsion) the following inequality [3] is valid:

$$\left|\frac{\boldsymbol{k}_s\cdot\boldsymbol{K}}{k_s\langle\bar{\varrho}(\boldsymbol{r})\rangle}\left(\frac{\partial\bar{\varrho}(\boldsymbol{r}')}{\partial\beta}\right)_{\beta=0}\right| \ll \left|\frac{2}{c_s}\left(\frac{\partial c_0(\boldsymbol{r}')}{\partial\beta}\right)_{\beta=0}\right|. \tag{4.18}$$

Thus B_p may be calculated, with the desired degree of accuracy, from the following formula:

$$B_p = \frac{k_s^4 P_0^2 V'}{16\pi^2} \langle(\delta\beta)^2\rangle \left[\frac{2}{c_s}\left(\frac{\partial c_0(\boldsymbol{r}')}{\partial\beta}\right)_{\beta=0}\right]^2. \tag{4.19}$$

Formula (4.16) then agrees with the relevant formula given in [5].

Substituting into (4.16), (4.19) the relations

$$I_0 = AP_0^2, \quad I_{sc} = A\langle|\Delta p_{sc}(\boldsymbol{r})|^2\rangle, \quad A = \text{const}, \tag{4.20}$$

we obtain, after integrating in all directions ($\boldsymbol{K}$ being the polar axis):

$$I_{sc} = I_0 \frac{k_s^4 V'}{4\pi r^2 K} \langle(\delta\beta)^2\rangle \left[\frac{2}{c_s}\left(\frac{\partial c_0(\boldsymbol{r}')}{\partial\beta}\right)_{\beta=0}\right]^2 \int_0^\infty x\gamma(x)\sin Kx\, dx, \tag{4.21}$$

where I_0 and I_{sc} denote the intensity of the incident and scattered waves, respectively. This relation can be regarded as an integral equation for the correlation function $\gamma(x)$. Using the Fourier integral transformation for odd function and formulae (2.13), (3.16) we finally obtain

$$\gamma(x) = \left[\int_0^\infty I_{sc} K^2 \frac{\sin Kx}{Kx} dK\right] \cdot \left[\int_0^\infty I_{sc} K^2 dK\right]^{-1}, \tag{4.22}$$

$$\omega = \text{const}, \quad r = \text{const}.$$

Formula (4.22) enables us to determine the correlation function $\gamma(x)$ from the angular distribution of the intensity of the wave scattered by a random isotropic nonviscous emulsion.

5. Final remarks

It has been shown that it is possible to determine the correlation function $\gamma(x)$ from the angular distribution of the scattered scalar potential. This may be done using formula (3.17). However, formula (3.17) has rather theoretical value. In contrast to formula (3.17) the basic results of sections 3 and 4 have a practical value. With the help of formulae (3.18) and (4.22), their value can be seen in the fact that those enable us to determine the correlation function $\gamma(x)$ from the appropriate measurements of the angular distribution of the intensity of the wave scattered by a random isotropic granular medium. The function $\gamma(x)$ which drops from 1 to 0 indicates the average extension of inhomogeneities. As a measure for their size we could adopt the value L_c of x for which $\gamma(x)$ becomes equal to $1/e$.

References

[1] L. A. Chernov, *Wave propagation in a random medium*, Dover — New York 1960.

[2] T. S. Chow, *Scattering of elastic waves in an inhomogeneous solid*, J. Acoust. Soc. Amer., **56**, 4, 1049–1051 (1971).

[3] A. S. Davydov, *Quantum mechanics*, PWN, Warsaw 1969 (in Polish), p. 378–381.

[4] P. Debye and A. M. Bueche, *Scattering by an inhomogeneous solid*, J. Appl. Phys., **20**, 6, 518–525 (1949).

[5] I. Malecki, *The "space sources" method of investigating ultrasonic wave propagation in grain media* (in Polish), Arch. Elektrotech., **5**, 546–697 (1956).

[6] C. L. Pekeris, *Note on scattering in an inhomogeneous medium*, Phys. Rev., **71**, 4, 268 (1947).

[7] M. Razavy, *Determination of the wave velocity in an inhomogeneous medium from the reflection coefficient*, J. Acoust. Soc. Amer., **58**, 5, 956–963 (1975).

Received on 10th September 1977

C H R O N I C L E

4-TH INTERNATIONAL CONFERENCE ON "ENVIRONMENTAL PROTECTION IN MECHANICAL ENGINEERING"

Györ, Hungary, April 11-13, 1978

The 4-th Conference, held April 11–13, 1978, at Györ (HPR), was dedicated to the problems connected with a broadly conceived environmental protection against harmful effects and consequences of production processes and technical, technological and communication equipment encountered chiefly in the mechanical engineering. The Conference was sponsored by the local section in Györ of the Hungarian Scientific Society of Mechanical Engineers in cooperation with the local section of the Hungarian Optical, Acoustical and Film Technical Society (OPAKFI), as also with the Board for the Matters of Natural Environment at Györ. Chairman of the Organizational Committee was Mr. J. Jambor, Secretary General Mr. E. Varga. The deliberations took place in the Cultural Centre Raba at Györ, Széchery Square 7.

Main themes involved three groups of problems:

I. Noise and vibration protection.

II. Water pollution protection.

III. Air pollution protection.

According to this program the deliberations took place simultaneously and independently in three sections designated with numbers I, II and III, respectively. The main aid of the Conference was:

— the discussion of actual and steadily growing threat to natural environment in three above mentioned fields caused by the intensive development and thus of the ever wider range of harmful effects of the mechanical engineering, communication and other related branches of engineering;

— a search for new methods, means and systems of the elimination or reduction of these effects or the protection of man against these effects.

The conference was attended by some 300 participants, including several scores of specialists from the following countries: England, Belgium, Czechoslovakia, Denmark, France, Yugoslavia, the German Democratic Republic, Poland and the Federal Republic of Germany. The Polish delegation consisting of seven persons representing the Polish Academy of Sciences, technical universities and institutes sponsored by respective ministries attended only the deliberations of the Section I, "Noise and vibration protection", delivering four lectures out of five included in the preliminary program. The number of lectures to be delivered in individual sections was the following: Section I — 29 lectures, Section II — 15 lectures, Section III — 10 lectures. The conference was begun by a plenary session at which Dr Tibor Bakács, Chairman of the Economic Committee and Labour Rights

Protection of the Hungarian Academy of Sciences delivered a lecture on the crucial subject: "Main problems of the environmental protection".

With view to the absence of Poland's representatives in the deliberations of the Sections II and III and the subject of interest of readers of "Archives of Acoustics", this report will only concern the deliberations of the Section I.

The Section held three sessions and ended its deliberation with a round-table conference summing up its proceedings. Lectures delivered:

1. G. Újsághy (Hungarian People's Republic), *Possibilities and limitations of the use of magnetic recording for making measurements in the field of noise and vibration protection.*

2. J. Kacprowski, J. Motylewski (Polish People's Republic), *Measurement of noise and acoustic diagnostics of machines.*

3. J. Braasch (Denmark), *New devices for measuring of noise.*

4. D. Zorič (Yugoslavia), *Aircraft noise: interdisciplinary aspects and ethic problems.*

5. J. Miazga (Polish People's Republic), *Noise of automative vehicles as a threat to man's natural environment.*

6. L. Czabalay (Hungarian People's Republic), *An analysis of methods for the evaluation of communication noise.*

7. F. Augusztinovicz, B. Buna (Hungarian People's Republic), *The method of control measurements of noise inside and outside of automotive vehicles.*

8. G. Pota (Hungarian People's Republic), *The efficiency of noise control inside and outside of buildings.*

9. L. Sárvári, T. Marjat, E. Kunos, J. Kovásc (Hungarian People's Republic), *The results of measurements of noise in new residential districts at Györ.*

10. V. Miklós (Hungarian People's Republic) *The effect of the design and exploitation conditions of power substations on the level of produced noise.*

11. B. Jost (France), *The evaluation of noise level in industrial buildings.*

12. T. Szentmártony (Hungarian People's Republic), *Damping of flow noise.*

13. H. Bauer (Federal Republic of Germany), *The noise produced by conventional power stations and the present state of its damping.*

14. J. Giergiel (Polish People's Republic), *Construction means and possibilities of reducing the noise of rotating machines.*

15. D. Sturm (Federal Republic of Germany), *The damping of noise by the use of shields on an example of three big-power blowers.*

16. V. Nösselt (Belgium), *The problem of the ratio of formant frequencies to fundamental frequency of noise in the light of measuring techniques.*

17. J. Kazimierczak (Polish People's Republic), *Noise of machines as a subject of investigations and results of work of their designer.*

18. K. Töpfer (German Democratic Republic), *The evaluation of new efficient calculation methods of noise insulation in rail vehicles.*

19. P. Tokarz (Polish People's Republic), *The effect of a working point on the noise characteristic of radial fans* (the lecture was not delivered).

20. L. Timár-Peregrin (Hungarian People's Republic), *Noise and vibrations of rotating electric machines and their identification by measurements.*

21. V. Stuchlik (Czechoslovakia), *Measurement and evaluation of noise for the purpose of health protection.*

22. W. Pollandt, H. Walter (German Democratic Republic), *Measurement of noise of mechanical vehicles in industrial plants and investigations on reducing the main noise sources.*

23. S. Spellenberg (Hungarian People's Republic), *Experiments on the complex individual noise protectors in case of a light or medium hearing impairment.*

24. H. G. Dieroff (German Democratic Republic), *Mechanism of hearing impairment caused by stationary and impulse noise on a work stand.*

25. E. Hochenburger, T. Katona, D. Martikány, O. Ribário, G. Váró (Hungarian People's Republic), *Importance of Earprotectometer in preventing the hearing impairment because of noise.*

26. G. Váró (Hungarian People's Republic), *Antivibrating rubber backings for machines in textile industry.*

27. J. Karucz (Hungarian People's Republic), *Previous and present results of measurements of noise damping in Metallurgical Plant at Őzd.*

28. M. Gabnai, E. Baross (Hungarian People's Republic), *Investigations of noise protection of workers in industrial plants exposed to the action of noise.*

29. L. Traegeman (German Democratic Republic), *Noise control in traffic* (film show).

It follows from this enumeration that 9 lectures, that is, about 32% of the total in Section I (Nos. 1, 2, 3, 6, 7, 11, 16, 20, 21) concerned new methods, systems and devices for the measurement and analysis of noise and vibrations; 7 lectures, that is, about 24% (Nos. 8, 12, 15, 18, 26, 27, 29) dealt with various systems and equipment for silencing the industrial noise sources; 5 lectures, that is, about 16% (Nos. 4, 5, 9, 13, 22) concerned the intensivity and harmfulness of industrial and traffic noise of different physical structures encountered in factories, residential buildings and districts; 4 lectures, that is, about 14% (Nos. 10, 14, 17, 19) stressed the role and importance of silent-running machines and equipment from the viewpoint of their designing; 4 lectures, that is, about 14% (Nos. 23, 24, 25, 28) were dedicated to the methods of preventing the hearing impairment caused by industrial noise by the use of individual protectors.

The scientific level of the lectures was not especially high since they were chiefly addressed to the industrial workers as future recipients and users of the methods, systems and equipment for the measurement of noise and vibrations and noise and vibrations control.

Nevertheless a number of conceptional, technical and constructional solutions in the field of the instrumentation for the purposes of the metrology of noise and vibrations and for the design of silent-running machines and equipment deserve attention because of their originality.

An important role played the round-table discussion which in a way has extended and supplemented the discussion following individual lectures and at the same time enabled mutual exchange of experience and information between representatives of various fields of science, technics and engineering who represented various European countries. This has permitted to estimate objectively the present state in this field of acoustics and point out new trends of development and formulate guidelines in realizing an efficient policy in the range of noise control.

The organization of the Conference deserves a high praise both as regards the provision of technical facilities for a simultaneous and efficient translation from Hungarian into two languages English and German and conversely and the value of informative material published in one volume containing the full texts of lectures delivered at three section in English and German. One copy of this publication is available in the Acoustic Library of the Institute of Fundamental Technological Research (Warszawa).

Janusz Kacprowski (Warszawa)